Social Evaluations in a Context of Demographic Changes: The Case of Sub-Saharan Africa

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Abstract

This paper focusses on how the progress of nations can be evaluated when populations differ in size, longevity and income distributions. The framework is applied to the (particular) demographic context of Sub-Saharan Africa (SSA). The findings indicate that the contribution of population size to social welfare depends on ethical considerations regarding the choice of a critical level above which a life is considered to be worth living (or social welfare improving). Length of life does not have a significant effect on social welfare prior to the demographic transition. SSA's demographic explosion over the last century has worsened social welfare for critical-level values greater than \$180 per year, i.e. roughly half the well-known dollar-a-day poverty line. This supports the often heard view that slowing down demographic growth in SSA may not only increase average living standards but may also raise overall social welfare.

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1 Introduction

Economic development is consistently linked to improved health outcomes and slower demographic growth. Developed countries have already passed the so-called critical point of a demographic transition. Beyond this critical point, populations are characterized by relatively high life expectancy along with a fertility rate that is equal to the natural population replacement rate. This demographic transition has also started to occur in the developing world (see Figure 1). In recent years, many developing countries have indeed experienced trends towards significantly lower fertility rates and increased life expectancy.

Sub-Saharan African (SSA) countries are by and large an exception to these trends among the least developed countries. SSA infant mortality rates remain high at 77 per thousand in 2010. The AIDS pandemic and several other geographically concentrated diseases such as malaria have constrained health improvements in the region. SSA fertility rates are high and overcompensate for these high mortality levels. Demographic growth in SSA is the highest in the world, at 2.5% per year in 2010.

Looking further back in time, SSA's last century has been characterized by an exploding population with limited improvements in income and life expectancy. The population has expanded tenfold between 1910 and 2010, at an average annual growth rate of 2.3%. In contrast, per capita GDP has grown at an annual average rate of only 0.8% over that period. Life expectancy at birth has doubled from 27 years in 1910 to 54 years in 2010, corresponding to an annual average increase of 0.7%.

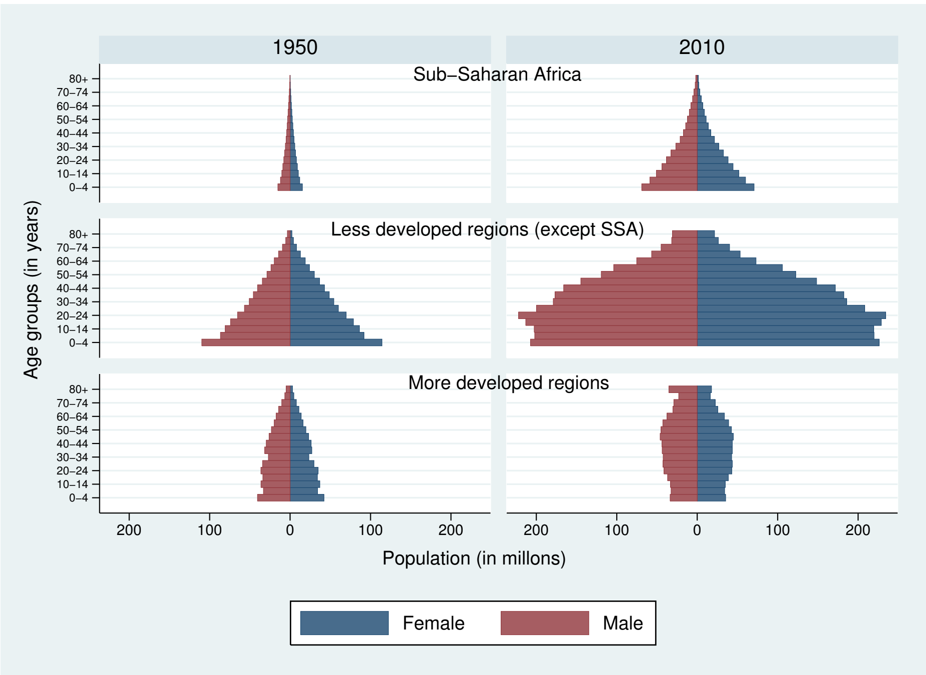

Figure 1: Demographic transition for the different regions of the World

How do these demographic and health trends affect social welfare in SSA? Answering this question is the main purpose of this paper. To do so, the paper *i*)establishes a dominance criterion for welfare comparisons when populations differ in size and in longevity, using intertemporal social evaluation functions developed in the literature; *ii*)performs temporal comparisons of social welfare in SSA during the last century by jointly considering changes in, and levels of, longevity, population size and incomes; and *iii*)evaluates the effects of changes in population size and longevity on social welfare in SSA.

Incorporating longevity and demographic variables into evaluations of social welfare is consistent with recent academic advances in development and welfare economics. The objectives of development policy have evolved along these lines over recent decades, shifting somewhat from the traditional objective of income and economic growth towards broader, and sometimes more sustainable, human development goals. Longevity and the distribution of welfare over time are crucial elements of human development, as argued below in the first UNDP human development report:

> "Human development is a process of enlarging people's choices. In principle, these choices can be infinite and change over time. But at all levels of development, the three essential ones are for people to lead a long and healthy life, to acquire knowledge and to have access to resources needed for a decent standard of living. If these essential choices are not available, many other opportunities remain inaccessible." (UNDP, 1990, p. 10).

In such a context, social evaluation principles can be set in a welfarist intertemporal framework and enable trade-offs between standards of living, longevity and population size. Performing intertemporal social comparisons in a welfarist framework amounts to ranking a two-dimensional matrix (individuals and time periods) of individual welfare defined across different social states. Given an intertemporal social evaluation function W, social state A is deemed better than the social state B in terms of welfare if and only if $W_A \geq W_B$. If the welfare function W is uniquely defined, this comparison is easily performed by comparing the value of the function for the two social states A and B. To agree on a unique social evaluation function is, however, an ambitious task; views differ widely as to how to value and to trade-off the importance of such things as the length of lives, the quantity of lives and the living standards enjoyed during these lives (the "quality" of these lives). It may thus be useful to perform dominance tests when comparing social states in such a context.

Kolm (1969), Atkinson (1970), Shorrocks (1983) and Kakwani (1984) provide foundations for atemporal welfare dominance in the case of fixed populations by using Dalton's population principle. Dalton's population principle stipulates that an income distribution and its r-times replication (for an arbitrary integer r) yield identical levels of social welfare: population size does not matter *per se* in social evaluation. Shorrocks (1983) and Kakwani (1984) in particular establish an equivalence between welfare dominance criteria based on average generalized utilitarianism (AGU) and a generalized-Lorenz-curve (GLC) dominance criterion — see below for more details. According to this, an income distribution second-order dominates a second one if the GLC of the first lies above the GLC of the latter.

Similar dominance criteria have been developed recently for the case of variable population sizes using critical-level generalized utilitarianism (CLGU) as an alternative principle to AGU. Trannoy and Weymark (2009) define a second-order dominance criterion based on CLGU through the use of *generalized concentration curves*. Duclos and Zabsonré (2010) develop alternative techniques that can order distributions of different population sizes at arbitrary orders of dominance and present methods for setting lower and upper bounds of critical levels in those dominance comparisons.

To the best of our knowledge, no dominance criteria have yet been provided for intertemporal CLGU[1]. This is the first objective of this paper. Having set a robust normative framework for assessing the lengths, quantities and qualities of lives, the second objective is to apply that framework to assess the evolution of social welfare in SSA. It is found that the contribution of population size to social welfare depends on the normative choice of a critical level — the point at which a life is considered to be worth living and social welfare improving. For instance, SSA's demographic growth over the last century has worsened social welfare for critical-level values greater than $180 per year, *i.e.*, roughly half the well-known dollar-a-day poverty line. Increases in life expectancy did not impact significantly the social welfare.

The rest of the paper is organized as follows. Section 2 describes briefly the dominance criteria found in the literature for atemporal social evaluations principles, considering in particular AGU and CLGU, and based on generalized Lorenz dominance and generalized concentration curve dominance. Section 3 defines multi-period critical-level utilitarian dominance and then derives multi-period dominance criteria. Section 4 describes the data and SSA's economic development and demographic transition. Intertemporal welfare comparisons are performed in Section 5 and Section 6 concludes.

[1] Duclos and Housseini (2013) do develop, however, procedures for constructing multi-period critical-level utilitarian functions.

2 Welfare dominance for average and critical-level utilitarian principles

2.1 Welfare dominance for average generalized utilitarianism

2.1.1 Average generalized utilitarian dominance

Let $u = (u_1, ..., u_n) \in \mathbb{R}^n$ be a distribution of atemporal utility among a population of size $n \in \mathbb{N}$. u_i will be interpreted later on as the income of individual i. The set of possible utility distributions is $\mathcal{U} = U_{n\in\mathbb{N}}\mathbb{R}^n$. Average generalized utilitarian social welfare functions are defined by the following social welfare function:

$$W_g^{AU}(u) = \frac{1}{n}\sum_{i=1}^{n} g(u_i). \tag{1}$$

where g is an increasing transformation $g : \mathbb{R} \longmapsto \mathbb{R}$ of utilities. The functions $W_g^{AU}(u)$ are said to be welfarist because they depend solely on the vector of individual utilities u. A specific case of $W_g^{AU}(u)$ is average utilitarianism, by which $g(u) = u$.

Average generalized utilitarianism satisfies Dalton's population principle, which stipulates that an income distribution and its r-fold replication (for an arbitrary integer r) yield identical social welfare. According to this principle, adding a new individual to an existing population increases social welfare if and only if his utility is greater than the average utility of the existing population (further discussion of this in Blackorby et al., 2005).

Dominance testing proceeds from the general form in (1) by positing properties that $g(\cdot)$ must obey. The property that is most commonly imposed is concavity. $g(\cdot)$ functions that are concave obey the well-known Pigou-Dalton transfer principle, by which a mean-preserving and rank-preserving transfer of utility from a better-off to a lesser-off individual must increase social welfare. Imposing concavity leads to criteria for "second-order" dominance. In the welfare economics tradition, the paper focuses on such second-order dominance techniques, although generalization to other orders of dominance are possible, as done for instance in Cockburn et al. (2012). The equivalence results shown below indicate how simple comparisons of dominance curves can establish unambiguous rankings over various sorts of atemporal, intertemporal, fixed-population and variable-population size second-order classes of social welfare functions.

Average generalized utilitarian dominance

Average generalized utilitarian dominance is defined by the quasi-ordering $\succeq^{AU}$ on the set of utility distributions $\mathcal{U}$ by considering the intersection of all average generalized utilitarian social welfare functions W_g^{AU} with $g(u)$ concave in u. For two given vectors of lifetime *utility u and u', u average-generalized-utilitarian dominates u' if and only if social welfare with u is greater than social welfare with u' as measured by any average generalized utilitarian function W_g^{AU} with concave g:*

$$u \succeq^{AU} u' \Leftrightarrow W_g^{AU}(u) \geq W_g^{AU}(u'). \tag{2}$$

The Lorenz curve ranks utility distributions by their level of inequality (Atkinson, 1970), thus ignoring the average level of utility. To take into account differences in average utility as well as differences in inequality, Shorrocks (1983) and Kakwani (1984) introduce the *generalized Lorenz curve (GLC)*, that is, the Lorenz curve multiplied by average utility. For a utility distribution $u \in \mathcal{U}$, let $u_\uparrow$ be the vector in which the components of u have been rearranged in a non-decreasing order. Let $p = \frac{k}{n}$ for some $k \in \{1, ..., n\}$. The GLC is the function $GL_u : [0, 1] \to \mathbb{R}$ defined as:

$$GL_u(p) = \frac{1}{n} \sum_{i=1}^{k} u_{\uparrow i}. \tag{3}$$

For non-integer values of k, $GL_u(p)$ is obtained by linear interpolation. $GL_u(p)$ gives the contribution of the poorest p% of the population to *per capita* income.

Numerical example: For a utility vector $u = (17, 23, 20, 30, 25, 12, 34, 45, 26, 32)$, the corresponding generalized Lorenz curve is as follows:

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$u_\uparrow$	12	17	20	23	25	26	30	32	34	45
$GL_u(p)$	1.2	2.9	4.9	7.2	9.7	12.3	15.3	18.5	21.9	26.4

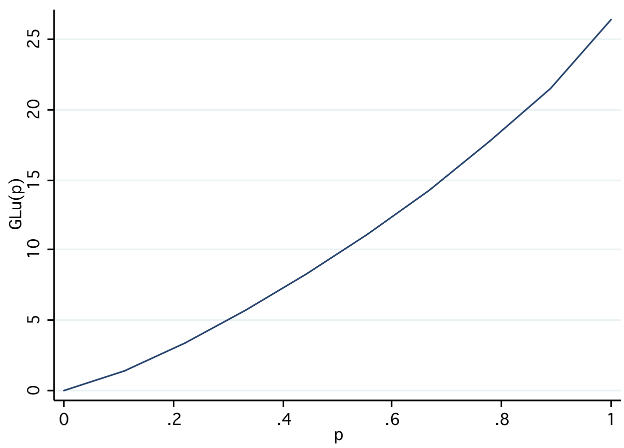

Figure 2: Example of a generalized Lorenz curve

Generalized Lorenz dominance

Generalized Lorenz dominance is defined by the quasi-ordering $\succeq^{GL}$ on the set of utility distributions $\mathcal{U}$ by comparing the GLC of utility distributions. For two given vectors of atemporal utility u and u', u generalized-Lorenz-dominates u' if and only if the GLC of u lies above the GLC of u'. Thus,

$$u \succeq^{GL} u' \Leftrightarrow GL_u(p) \geq GL_{u'}(p) \qquad for \quad all \quad p \in [0, 1]. \tag{4}$$

Shorrocks (1983) and Kakwani (1984) establish equivalence between average generalized utilitarian dominance and generalized Lorenz dominance.

The Kakwani-Shorrocks Theorem:
For all $u, u' \in \mathcal{U}$, $u \succeq^{GL} u' \Leftrightarrow u \succeq^{AU} u'$.

2.2 Welfare dominance for critical-level generalized utilitarianism

2.2.1 Critical-level generalized utilitarian dominance

Critical-level generalized utilitarianism (CLGU) was introduced by Blackorby and Donaldson (1984) to overcome some of the limitations of total (or classical) utilitarianism and average utilitarianism. Total utilitarianism leads to what is referred to in the literature as the *repugnant conclusion* (Parfit, 1984). Total utilitarianism deems a sufficiently large population to be better than a much smaller population, even when the larger population has a very low average utility. Average utilitarianism implies that a one-person society with a high average utility dominates any other society with lower average utility, regardless of how large the second society is. The CLGU principle is introduced to avoid these undesirable results and to provide an arguably more appropriate criterion to value the welfare of populations with different sizes. A CLGU social welfare function can be defined as:

$$W_{\alpha,g}^{CL}(u) = \sum_{i=1}^{n} [g(u_i) - g(\alpha)] \tag{5}$$

for $u \in \mathbb{R}^n$ where g is an increasing utility transformation $g : \mathbb{R} \longmapsto \mathbb{R}$ and α is the critical level utility, namely, the level of utility of an individual whose contribution to social utility is neither positive nor negative. α can also be viewed as a normative threshold for valuing lives and for whether a society will benefit from the addition of a new person. The CLGU social welfare function in (5) is the sum of transformed utilities net of the same transformation of the critical level.

CLGU dominance
The CLGU dominance is defined by the quasi-ordering $\succeq_{\alpha}^{CL}$ on the set of utility distributions $\mathcal{U}$ by considering the intersection of all CLGU social welfare functions with concave g. For two vectors of utility u and u', u CLGU dominates u' if and only if social welfare with u is greater than social welfare with u' as measured by any CLGU function $W_{\alpha,g}^{CL}$ with concave g:

$$u \succeq_{\alpha}^{CL} u' \Leftrightarrow W_{\alpha,g}^{CL}(u) \geq W_{\alpha,g}^{CL}(u'). \tag{6}$$

Note that CLGU is equivalent to classical utilitarianism if $g(\alpha) = 0$. With identical population sizes, critical-level generalized utilitarian dominance is equivalent to average generalized utilitarian dominance.

2.2.2 Critical-level generalized concentration curve dominance

In the manner of the generalized Lorenz curve for average generalized utilitarianism, Trannoy and Weymark (2009) define a *generalized concentration curve (GCC)*, defined as follows:

> For any utility distribution, a generalized concentration curve plots the sum of the utility of the t individuals with the smallest utilities against t, using linear interpolation so that the curve is defined for non-integer values of t (Trannoy and Weymark, 2009, p. 271).

Formally, for $u \in \mathcal{U}$, the GCC is the function $GC_u : [0, n] \to \mathbb{R}$ defined for $t \in \{1, ..., n\}$ as

$$GC_u(t) = \sum_{i=1}^{t} u_{\uparrow i}. \tag{7}$$

For non-integer values of t, $GC_u(t)$ is obtained by linear interpolation.

Numerical example: For the utility vector $u = (17, 23, 20, 30, 25, 12, 34, 45, 26, 32)$ of the preceding example, the generalized concentration curve is given by:

t	1	2	3	4	5	6	7	8	9	10
$u_{\uparrow}$	12	17	20	23	25	26	30	32	34	45
$GC_u(t)$	12	29	49	72	97	123	153	185	219	264

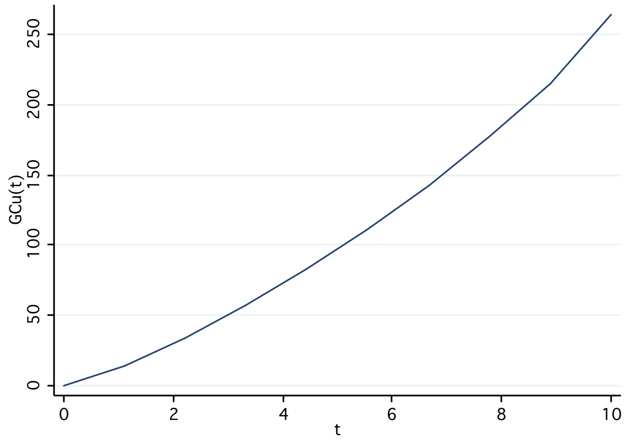

Figure 3: Example of a generalized concentration curve

For generalized Lorenz dominance, the Dalton population principle is used to replicate populations and to deal with different population sizes. For CLGU dominance, Trannoy and Weymark (2009) consider *augmented* utility vectors. For all $u \in \mathcal{U}$, $\alpha \in \mathbb{R}$ and $\overline{n} \in \mathbb{N}$, we obtain an augmented utility vector $u_{\alpha,\overline{n}} = (u, \alpha\mathbf{1}_{\overline{n}})$, where $\mathbf{1}_{\overline{n}}$ is the vector of *1*s in $\mathbb{R}^{\overline{n}}$. $u_{\alpha,\overline{n}}$ is thus of size $n + \overline{n}$ with $\overline{n}$ values of α.

Critical-level GCC dominance

Critical-level GCC dominance is defined by the quasi-ordering $\succeq_{\alpha}^{GC}$ on the set of utility distributions $\mathcal{U}$ and by comparing the GCC of augmented utility distributions. For all $u, u' \in \mathcal{U}$, let $\overline{n}(u, u') = 0$ if $n \geq n'$ and $\overline{n}(u, u') = n' - n$ otherwise; u critical-level GCC dominates u' if and only if the GCC of $u_{\alpha,\overline{n}(u,u')}$ lies above the GCC of $u'_{\alpha,\overline{n}(u',u)}$ across the entire distribution:

$$u \succeq_{\alpha}^{GC} u' \Leftrightarrow GC_{u_{\alpha,\overline{n}(u,u')}}(t) \geq GC_{u'_{\alpha,\overline{n}(u',u)}}(t) \quad for \quad all \quad t \in [0, max\{n, n'\}] \tag{8}$$

After having defined the GCC, Trannoy and Weymark (2009) establish the following equivalence between CLGU dominance and GCC dominance:

The Trannoy-Weymark Theorem:
For any $\alpha \in \mathbb{R}$, for all $u, u' \in \mathcal{U}$, $u \succeq_{\alpha}^{GC} u' \Leftrightarrow u \succeq_{\alpha}^{CL} u'$.

3 Welfare dominance for multi-period CLGU

Evaluating public policies often involves comparing social states of the world in which populations differ in size and longevity. This requires social evaluation principles to be set up in a normative framework that is sensitive both to the number and length of lives. Intertemporal CLGU in its multi-period version was introduced by Duclos and Housseini (2013) to make such analysis possible and to overcome some of the limits of the non-temporal CLGU presented above, notably its failure to avoid a *temporally repugnant conclusion* and the fact that it must satisfy the *critical-level temporal consistency* property (further details in Duclos and Housseini, 2013). Our objective in this section is to establish a dominance criterion for multi-period CLGU that is similar to both Lorenz and concentration curve dominances.

3.1 Multi-period CLGU dominance

Performing social ranking when populations differ in size and longevity amounts to comparing social states of the world defined by a matrix of utilities in two dimensions: individual and time period. Thus, a social state $x \in \mathcal{X}$ is defined by a matrix of utilities $U \in \mathcal{M}_u$ that gives periodic utilities rather than vectors of lifetime utility for different individuals. $U = \{u_{i,j}\}_{i \in \mathcal{N}; j \in \mathcal{T}}$, where $\mathcal{N} = \{1, 2, ..., n\}$ is the set of individuals and $\mathcal{T} = \{1, 2, ..., T\}$ is the time frame analyzed. Thus, the total number of individuals is n and the last period of evaluation is T. The time periods are simply a placeholder and can represent any desired length of time.

Basically, individuals are born on different dates and have different lengths of life. Period 1 thus corresponds to the date of birth of the first person born and the last period T is the date of death of the last person alive. Thus, some individuals have not been born yet at the beginning of the period of analysis and others die before T. The utility remains blank in the matrix if the individual is not alive in the corresponding period. A sample utility matrix follows:

Persons\periods	1	2	3	.	.	$T-1$	T
1		$u_{1,2}$	$u_{1,3}$	..	..	$u_{1,T-1}$	
2	$u_{2,1}$	$u_{2,2}$	$u_{2,3}$				
3				$u_{3,.}$	$u_{3,.}$	$u_{3,T-1}$	$u_{3,T}$
.	$u_{.,1}$	$u_{.,2}$	$u_{.,3}$	$u_{.,.}$	$u_{.,.}$		
.		$u_{.,2}$	$u_{.,3}$				
n	$u_{n,1}$	$u_{n,2}$	$u_{n,3}$	..	..	$u_{n,T-1}$	$u_{n,T}$

The multi-period CLGU principle for such a utility matrix is defined by the following social welfare function:

$$W_{\alpha,g}^{PCL}(U) = \sum_{i=1}^{n} \sum_{t=s_i+1}^{s_i+T_i} [g(u_{it}) - g(\alpha)] \tag{9}$$

for $U \in \mathcal{M}_u$, and where s_i is the date of birth of person i, $s_i + T_i$ his date of death, g is the utility transformation $g : \mathbb{R} \longmapsto \mathbb{R}$ defining the attitude of the social planner towards inequality in the utility distribution and α is the periodic critical-level utility. α is a level of periodic utility enjoyed by an arbitrary person i in an additional period of life that does not alter the social evaluation function. Thus, an additional period of life increases social welfare if the additional utility is above the periodic critical level, and conversely is welfare-decreasing if the utility is below α. Building on this multi-period CLGU social welfare function, we define the multi-period CLGU dominance as follows.

Multi-period CLGU dominance

The multi-period CLGU dominance is defined by the quasi-ordering $\succeq_{\alpha}^{PCL}$ on the set of utility matrices $\mathcal{U}$ by considering the intersection of all multi-period CLGU social welfare functions $W_{\alpha,g}^{PCL}$ with concave g. For two given utility matrices U and U', U multi-period CLGU dominates U' if and only if the social welfare with U is greater than the social welfare with U' as measured by any multi-period CLGU function $W_{\alpha,g}^{PCL}$ with concave g:

$$U \succeq_{\alpha}^{PCL} U' \Leftrightarrow W_{\alpha,g}^{PCL}(U) \geq W_{\alpha,g}^{PCL}(U'). \tag{10}$$

3.2 Multi-period critical-level generalized concentration curve dominance

The objective here is to define a dominance curve from a utility matrix that provides the same social ranking as the one obtained using a multi-period CLGU social evaluation function. To do this, we first introduce the concept of α *one-person equivalent population* and define the individual version of a generalized concentration curve (GCC) that will be used to rank two social states with different population sizes and lengths of life.

The α one-person equivalent population

To deal with populations of different sizes, each population is represented by an equivalent population that has the same level of social welfare and only one living person, the *one-person equivalent population*, defined as follows:

Definition 1. *Let $U = \{u_{i,j}\}_{i \in \mathcal{N}; j \in \mathcal{T}}$ be a utility matrix defined for a population P,* ***the α one-person equivalent population (OPEP)*** *is defined as a fictive one-person population $\widetilde{P}$ where the only person alive has a notional lifespan equal to the sum of lifespans* *of individuals in* P*,* $\widetilde{T} = T_1 + T_2 + ... + T_n$*, and a utility vector of* $\widetilde{u} = \{\{u_{1,j}\}_{j\in\mathcal{T}_1} \cup \{u_{2,j}\}_{j\in\mathcal{T}_2} \cup ... \cup \{u_{n,j}\}_{j\in\mathcal{T}_n}\}$*, such that* P *and* $\widetilde{P}$ *are deemed equally good by any multi-period CLGU social welfare function* $W^{PCL}_{\alpha,g}$*.*

Given the definition of $\widetilde{P}$, *temporal anonymity* and *indifference for unfragmented lives* of $W^{PCL}_{\alpha,g}$(established in Duclos and Housseini, 2013) both ensure that P and $\widetilde{P}$ are ranked as equally good by $W^{PCL}_{\alpha,g}$.

Example: Let P be a population whose utility matrix is given by:

5	7	3		
		13	13	9
12	10	9	11	

The utility vector of its OPEP $\widetilde{P}$ is then given by: $\widetilde{u} = (5, 7, 3, 13, 13, 9, 12, 10, 9, 11)$.

The temporal generalized concentration curve

The purpose of the temporal generalized concentration curve is to provide a graphical criterion to rank individuals using their distribution of utility over time. Thus, this curve can only be defined when knowing preferences and attitudes towards the temporal distribution of utility. To do this, we build on the literature of intertemporal welfare, poverty dynamics and lifetime poverty measurement (Bossert et al., 2010; Calvo and Dercon, 2009; Chakravarty et al., 1985; Foster, 2009; Hoy et al., 2010; Maasoumi and Zandvakili, 1986, 1990; Rodgers and Rodgers, 1993; Salas and Rabadan, 1998). In the lifetime poverty literature, the *chronic poverty axiom* is mostly used to characterize the measurement of lifetime poverty (Bossert et al., 2010; Foster, 2009; Hoy et al., 2010; Rodgers and Rodgers, 1993). Briefly, the chronic poverty axiom stipulates an aversion to utility being unequally distributed over time (temporal inequality). This suggests that an individual with a more equal distribution of utility over time is deemed better off than an individual with a less equal distribution of utility over time, everything else being the same. To account for this aversion to unequally distributed multi-period utility over time, the literature on intertemporal welfare measurement usually postulates a concave welfare function with respect to multi-period utilities (Maasoumi and Zandvakili, 1986, 1990; Salas and Rabadan, 1998). This preference for an equal distribution of utility across time is also consistent with consumer preferences for consumption smoothing over time. It is generally recognized in consumer theory that an agent's current consumption is determined by lifetime income rather than current income (Deaton and Paxton, 1994; Friedman, 1957). Given all this, we postulate a second order dominance curve called *the temporal generalized concentration curve* for ranking temporal distributions of utility. Recall that second order dominance corresponds to a concave inequality-averse welfare function to rank individuals. The temporal generalized concentration curve is defined as follows.

Definition 2. *For any temporal utility distribution, the* ***temporal generalized concentration curve (TGCC)*** *plots the sum of the multi-period utilities in the* t *periods with the* t *smallest utilities against* t*, using linear interpolation to define the curve for non-integer values of* t*.*
Formally, for a temporal utility distribution $u \in \mathbb{R}^T$*, the TGCC is the function* $TGC_u : [0, T] \to \mathbb{R}$*, defined as follows for* $t \in \{1, ..., T\}$*:*

$$TGC_u(t) = \sum_{i=1}^{t} u_{\uparrow i} \tag{11}$$

and for non-integers values of t*,* $TGC_u(t)$ *is obtained by linear interpolation.*

Numerical example: Let $u = (17, 33, 14, 32, 25, 18, 34, 37, 26, 15)$ be the temporal distribution of utility of person i. His temporal generalized concentration curve is then:

t	1	2	3	4	5	6	7	8	9	10
$u_{\uparrow}$	14	15	17	18	25	26	32	33	34	37
$GC_u(t)$	14	29	46	64	89	115	147	180	214	251

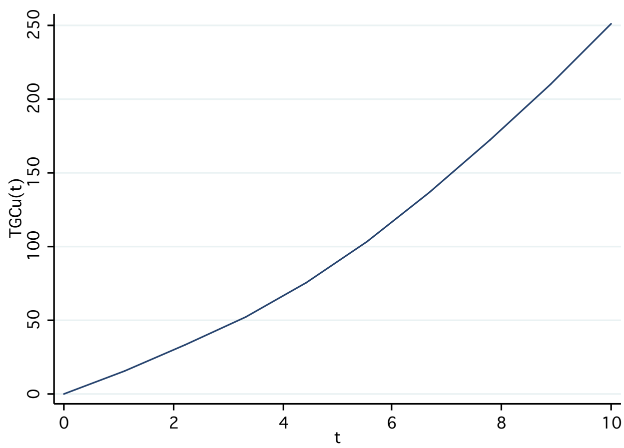

Figure 4: Example of a temporal generalized concentration curve

As in Trannoy and Weymark (2009), we define the *augmented temporal utility* vector for a temporal distribution of utility in order to be able to compare temporal generalized concentration curves defined for individuals with differing longevity.

The *augmented temporal utility* vector is defined as follows: for a temporal utility vector $u \in \mathcal{M}_u$, $\alpha \in \mathbb{R}$ and $\overline{T} \in \mathbb{N}$, the augmented temporal utility vector $u_{\alpha,\overline{T}} = (u, \alpha \mathbf{1}_{\overline{T}})$ where $\mathbf{1}_{\overline{T}}$ is the vector of *1*s in $\mathbb{R}^{\overline{T}}$.

Critical-level temporal GCC dominance

The critical-level temporal GCC dominance is defined by the quasi-ordering $\succeq_{\alpha}^{TGC}$ on the set of temporal utility vectors $\mathcal{U}$ by comparing the TGCC of augmented temporal utility vectors. For all $u, u' \in \mathcal{U}$, let $\overline{T}(u,u') = 0$ if $T \geq T'$ and $\overline{T}(u,u') = T' - T$ otherwise; u critical-level TGCC dominates u' if and only if the TGCC of $u_{\alpha,\overline{T}(u,u')}$ lies above the TGCC of $u'_{\alpha,\overline{T}(u',u)}$:

$$u \succeq_{\alpha}^{TGC} u' \Leftrightarrow TGC_{u_{\alpha,\overline{T}(u,u')}}(t) \geq TGC_{u'_{\alpha,\overline{T}(u',u)}}(t) \quad for \quad all \quad t \in [0, max\{T, T'\}] \qquad (12)$$

Note that if $g(\alpha) = 0$, the multi-period CLGU principle leads to intertemporal classical utilitarianism (developed by Blackorby et al., 1996a), and for populations of the same size where individuals have the same lengths of life, the critical-level generalized utilitarian dominance is equivalent to intertemporal average generalized utilitarian dominance that is defined by the following value function:

$$W^{IA}(U) = \frac{1}{nT} \sum_{i=1}^{n} \sum_{t=s_i+1}^{s_i+T} g(u_{it}) \qquad (13)$$

We use this intertemporal social evaluation function to define the intertemporal average generalized utilitarian dominance on the set of utility matrices $\mathcal{M}_u$.

Intertemporal average generalized utilitarian dominance

Intertemporal average generalized utilitarian dominance is defined by the quasi-ordering $\succeq^{IA}$ on the set of utility matrices $\mathcal{M}_u$ by considering the intersection of all intertemporal average generalized utilitarian social welfare functions. For two given utility matrices U and U', U intertemporally average-generalized-utilitarian dominates U' if and only if the social welfare with U is greater than the social welfare with U' as measured by any intertemporal average generalized utilitarian function W^{IA} with concave g:

$$U \succeq_{\alpha}^{IA} U' \Leftrightarrow W^{IA}(U) \geq W^{IA}(U'). \qquad (14)$$

We also introduce intertemporal generalized Lorenz dominance, which provides the same social ranking as that obtained using multi-period critical-level generalized concentration curve dominance when population size and longevity are fixed. The intertemporal generalized Lorenz dominance is defined as follows:

Intertemporal generalized Lorenz dominance

Intertemporal generalized Lorenz dominance is defined by the quasi-ordering $\succeq^{IGL}$ on the set of utility matrices $\mathcal{M}_u$ by comparing the intertemporal GLC of the OPEP utility distributions. For two given utility matrices U and U', U intertemporally generalized-Lorenz dominates U' if and only if the GLC of the temporal utility distribution $\widetilde{u}$ always lies above the GLC of the temporal distribution $\widetilde{u}'$:

$$U \succeq^{IGL} U' \Leftrightarrow GL_{\widetilde{u}}(t) \geq GL_{\widetilde{u}'}(t) \qquad for \quad all \quad t \in [0,1]. \tag{15}$$

3.3 Dominance criteria for multi-period CLGU: An equivalence theorem

After having defined multi-period CLGU dominance and critical-level temporal GCC dominance, the objective in this section is to establish equivalence between the two criteria such that the temporal GCC can be used for empirical applications. As argued in previous sections, comparisons by dominance curves are necessary given the arbitrariness of the utility transformation g in the definition of the multi-period CLGU function $W^{PCL}_{\alpha,g}$. We show here that for any critical-level α and any symmetric, increasing and concave function g, the quasi-ordering of matrices of multi-period utilities obtained using $W^{PCL}_{\alpha,g}$ is equivalent to that obtained using the critical-level temporal GCC dominance for the same value of α.

To do this, we first establish, in the following lemma, equivalence between intertemporal average generalized utilitarian dominance and intertemporal generalized Lorenz dominance.

Lemma 1. *For all $U, U' \in \mathcal{M}_u$, $\widetilde{u} \succeq^{IGL} \widetilde{u}' \Leftrightarrow U \succeq^{IA} U'$*

Proof. The result follows directly from i) the additive separability of W^{IA} that allows us to view the temporal distributions $\widetilde{u}$ and $\widetilde{u}'$ as distributions of individual lifetime utilities and ii) the Kakwani-Shorrocks theorem. □

We use the multi-period critical level principle to deal with populations of variable longevity and size, and to establish the equivalence between multi-period CLGU dominance and critical-level temporal GCC dominance formulated in the following theorem.

Theorem 1. *For any $\alpha \in \mathbb{R}$, for all $U, U' \in \mathcal{M}_u$, $U \succeq^{PCL}_{\alpha} U' \Leftrightarrow \widetilde{u} \succeq^{TGC}_{\alpha} \widetilde{u}'$.*

3.4 Multi-period critical-band generalized utilitarian dominance

Population critical-band utilitarianism was introduced by Blackorby et al. (1996b) to overcome some of the limitations of critical-level utilitarianism addressed by Broome (1992, 2004, 2007), the fixed critical level itself being criticized. Broome noted that critical-level utilitarianism also leads to a form of the Parfit repugnant conclusion that is commonly called the α-repugnant conclusion. As mentioned by Blackorby et al. (1996b):

> According to CLU, any social state with an average utility above the critical level is inferior to another state with a suitably large population and an average utility that is just above the critical level (Blackorby et al., 1996b).

This suggests that the critical-level utility should not be extremely low, but a higher value would prevent the existence of individuals whose lifetime utilities are just below it when the welfare of the existing population is unaffected. This leads Broome to reject the fixed critical level principle. Critical-band utilitarianism was developed to overcome this criticism. Rather than imposing a single value for the critical level, it considers a range of critical levels. The principle stipulates that an additional person with a lifetime utility that is above the upper bound of the range of critical levels is welfare-increasing, while an additional person with a lifetime utility that is below the lower bound of the range is welfare-decreasing. The welfare impact is indeterminate when the lifetime utility of the new person falls within the range of critical levels. Thus, for an interval of critical levels $[\underline{\alpha}, \overline{\alpha}]$, critical-band utilitarianism is defined by the following value function:

$$W^{CL}_{\alpha,g}(u) = \sum_{i=1}^{n}[g(u_i) - g(\alpha)], \quad \alpha \in [\underline{\alpha}, \overline{\alpha}] \tag{16}$$

The above arguments in favour of the critical-band population principle also apply to multi-period critical-level utilitarianism, developed to overcome the temporal repugnant conclusion. Indeed, the choice of a fixed value for the multi-period critical level could hardly provide satisfactory results. Choosing a low multi-period critical level may lead to the α-repugnant conclusion, while too high a value for the multi-period critical level in the social evaluation function may prevent the extension of a life that is worth living. Furthermore, the critical-band principles provide more robust social rankings given that they are less sensitive to the choice of a specific critical level within a consensual interval. Because of this, in this paper we also consider the multi-period critical-band generalized utilitarianism defined by the following value function:

$$W^{PCL}_{\alpha,g}(U) = \sum_{i=1}^{n}\sum_{t=s_i+1}^{s_i+T_i}[g(u_{it}) - g(\alpha)], \quad \alpha \in [\underline{\alpha}, \overline{\alpha}] \tag{17}$$

with $[\underline{\alpha}, \overline{\alpha}]$ being the range of multi-period critical levels.

We use this social evaluation principle to define multi-period critical-band generalized utilitarian dominance as follows:

Multi-period critical-band generalized utilitarian dominance

Multi-period critical-band $[\underline{\alpha}, \overline{\alpha}]$ generalized utilitarian dominance is defined by the quasi-ordering $\succeq^{PCB}_{[\underline{\alpha},\overline{\alpha}]}$ on the set of utility matrices $\mathcal{U}$ by considering the intersection of all multi-period CLGU social welfare functions for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. For two given utility matrices U and U', U multi-period critical-band-$[\underline{\alpha}, \overline{\alpha}]$-generalized-utilitarian dominates U' if and only if the social welfare with U is greater than the social welfare with U' as measured by any multi-period critical-level generalized utilitarian function $W^{PCL}_{\alpha,g}$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ and for all concave g:

For any $\underline{\alpha}, \overline{\alpha} \in \mathbb{R}$, and for all $U, U' \in \mathcal{M}_u$,

$$U \succeq^{PCB}_{[\underline{\alpha},\overline{\alpha}]} U' \Leftrightarrow U \succeq^{PCL}_{\alpha} U' \qquad \forall \alpha \in [\underline{\alpha}, \overline{\alpha}]. \tag{18}$$

For the purpose of empirical applications, the following corollary establishes equivalence between multi-period critical-band utilitarianism, multi-period critical-level utilitarianism and the temporal generalized concentration curve.

Corollary 1. *For any $\underline{\alpha}, \overline{\alpha} \in \mathbb{R}$, and for all $U, U' \in \mathcal{M}_u$, the following conditions are equivalent:*

1. $U \succeq^{PCB}_{[\underline{\alpha},\overline{\alpha}]} U'$
2. $\left[U \succeq^{PCL}_{\underline{\alpha}} U' \quad \text{and} \quad U \succeq^{PCL}_{\overline{\alpha}} U'\right]$, *and*
3. $\left[\widetilde{U} \succeq^{TGC}_{\underline{\alpha}} \widetilde{U}' \quad \text{and} \quad \widetilde{U} \succeq^{TGC}_{\overline{\alpha}} \widetilde{U}'\right]$.

4 Data and descriptive analysis

Our empirical analysis relies on three types of data: i)the annual distributions of income among individuals from 1820 to 2010, ii)demographic and health data, such as age structures of the population and life expectancies by age in each period and iii)income transition matrices from one period to another. We use these different data to estimate the income distributions, population sizes and structures and life expectancies by age in SSA between 1910 and 2010. Data on income distributions are from Bourguignon and Morrisson (2002), who provide historical data on income distributions for the different regions of the world for the period 1820-1992. We extended this dataset to 2010 by using the growth rates of per capita income published by the World Bank (2013) and by assuming, for simplicity, that inequality levels have remained unchanged between 1992 and 2010. Bourguignon and

Morrisson (2002)'s data are in the form of "grouped" income distributions by deciles. We regenerate samples of individual-level microdata for the different country groups of SSA, which sum to a total of 46 countries. The regeneration of samples is done by means of Shorrocks and Wan (2009)'s algorithm, which makes it possible to recreate individual-level microdata from the aggregated data[2]. We generated a sample of 1,000 observations for each of the four country groups and for every year from 1820 to 2010. Demographic data are from the Population Division of the UN Department of Economic and Social Affairs (2013), which provides the age structures of the world population for different regions and countries between 1950 and 2010. Life expectancies by age are estimated by combining historical data on life expectancy at birth provided by Bourguignon and Morrisson (2002) and the World Life Tables obtained from the World Health Organization, World Health Statistics (2012). Using these different datasets, the following sub-sections perform a descriptive analysis of changes in income distributions, life expectancy and population size in SSA over the last century.

4.1 Economic development, longevity and population size in Sub-Saharan Africa over the last century

The last century in Sub-Saharan Africa has been characterized by explosive population growth along with more limited improvements in income and life expectancy. Between 1910 and 2010, per capita GDP growth has averaged 0.8% per year. The population of the region grew tenfold over this timeframe, an average of 2.3% per year. Life expectancy at birth rose from 27 years in 1910 to 54 years in 2010, an average increase of 0.7% per year.

Figure 5: Life expectancy, population size and GDP *per capita* in Sub-Saharan Africa: levels (left panel) and decadal growth rates (right panel)

Two important questions arise from these empirical features. The first involves the nature of the relationship between population size, life expectancy and economic perfor-

[2] This was performed using the Stata DASP package; see Araar and Duclos (2007).

mance, and the second is to find out how population size and life expectancy affect social welfare. Many authors have explored the first question since the time of Malthus, but the debate remains unresolved. Some authors have recently suggested that these relationships depend on the nature of the economy being studied. For instance, Weil and Wilde (2009) found that the effect of population size on economic performance depends on the elasticity of substitution between land and other economic factors. For countries where this elasticity is low (developing countries), population size penalizes economic performance (Malthusian view), whereas the Malthusian view is not relevant in countries with a high elasticity of substitution (developed countries). The effect of life expectancy on per capita income is also ambiguous: Cervellati and Sunde (2009) found that life expectancy does not have a clear impact on economic development until the onset of the demographic transition, but that this effect is positive afterwards.

These developments make it more useful for our empirical application to focus on the second goal mentioned above: to determine the effect of population size and life expectancy on social welfare. The objective here is to show how the measured level of development differs when life expectancy and population size are considered in addition to living standards. In this regard, we first examine in the next sections the temporal evolution of each of our three variables of interest: income distribution, length of life and population size.

4.2 Evolution of income distributions in Sub-Saharan Africa

Figure 6 and figures 13, 14 and 15 (in the appendix) present a general description of income distributions in SSA over the 1910-2010 period. They provide a robust ordering of income distributions in terms of poverty, inequality and social welfare. Despite the explosion of population and increased inequality (Figure 13 in the appendix), all the social ordering criteria show a continous improvement of welfare in SSA between 1910 and 2010. Figure 6 compares 1910, 1960 and 2010 using the quantile curve that corresponds to a first-order dominance criterion for ranking income distributions. The quantile curve dominance is defined such that a distribution A dominates another distribution B if and only if, for all percentiles p, the income quantile $Q_A(p) = F_A^{-1}(p)$ is greater than $Q_B(p) = F_B^{-1}(p)$, where F refers to the cumulative income distribution function. If populations A and B have the same size, A dominates B according to this criterion if and only if for any individuals i_A and i_B of the same rank (in their respective income distributions), i_A is always richer than i_B. Thus, quantile curve dominance is stronger than the second order dominance criteria such as the generalized Lorenz dominance and generalized concentration dominance.

Therefore, as expected, the generalized Lorenz dominance confirms the improvement of incomes over the 1910-2010 period (Figure 14 in the Appendix). Recall that this social ranking criterion is equivalent to using an average generalized utilitarian function for

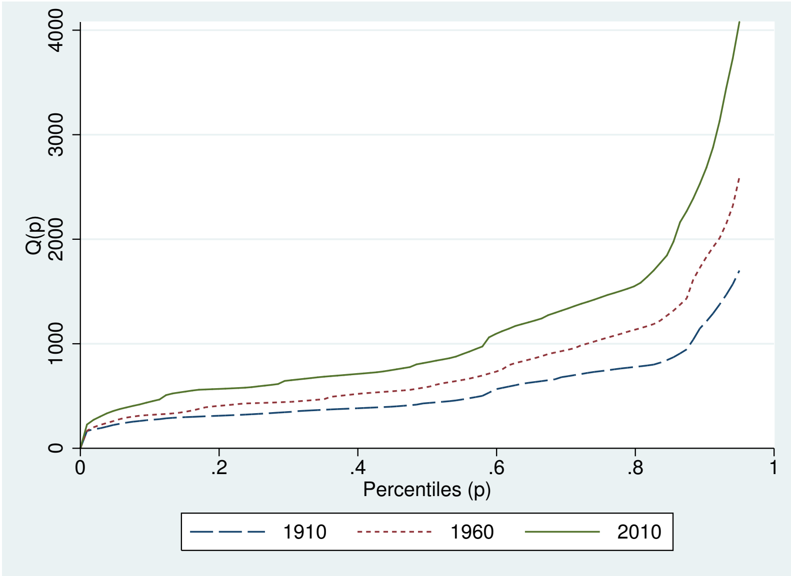

Figure 6: Quantile curves of income distributions in SSA over the last century

ranking income distributions. Thus, even if the growth of per capita income was combined with a rise in inequality, social welfare in the region should be deemed to have significatively increased over the last century. This indicates that the large increase of per capita income observed over the studied period has largely compensated (see Figure 14) the increase of inequality (see Figure 13) in income distribution.

Finally, the critical-level population principle also shows an improvement of social welfare in SSA between 1910 and 2010 for all values of critical-level $\alpha \in [0, 360]$, as illustrated by the critical-level generalized concentration curves plotted in Figure 15 of the Appendix (abstracting from differences in life expectancy). From an ethical perspective, a critical-band of $[0, 360]$ appears to be reasonable given that \$360 corresponds approximately to the well-known dollar-a-day poverty line. A greater critical-level can lead to a decrease in social welfare since 2010 would not then dominate 1910. The next section presents in further details the demographic patterns associated with these changes in income distributions.

4.3 Demographic transition in Sub-Saharan Africa

Figure 7 provides the temporal evolution of age structures for the different regions of the world. Developed regions have already passed the so-called critical point of demographic transition. Beyond this critical point, populations are characterized by relatively high life expectancy along with a low fertility rate which is equal to the natural population replacement rate. This results in an increased proportion of older people and a population more homogenously distributed among the different age groups. This demographic transition has also been happening in the developing world. In recent years, many developing countries have indeed experienced trends towards significantly lower fertility rates and increased life expectancy. Figure 7 clearly shows that in 2010, less developed regions (excluding SSA) have an age structure similar to that of developed regions in 1950. This points out a delay of approximatively half of a century between developed regions and less developed regions in the process of demographic transition. A large part of the population (45%) in less developed regions (excluding SSA) is young (aged less than 24 years). This age structure is changing gradually and the demographic transition is expected to be completed soon for these regions, which include most of the countries in Asia and Latin America.

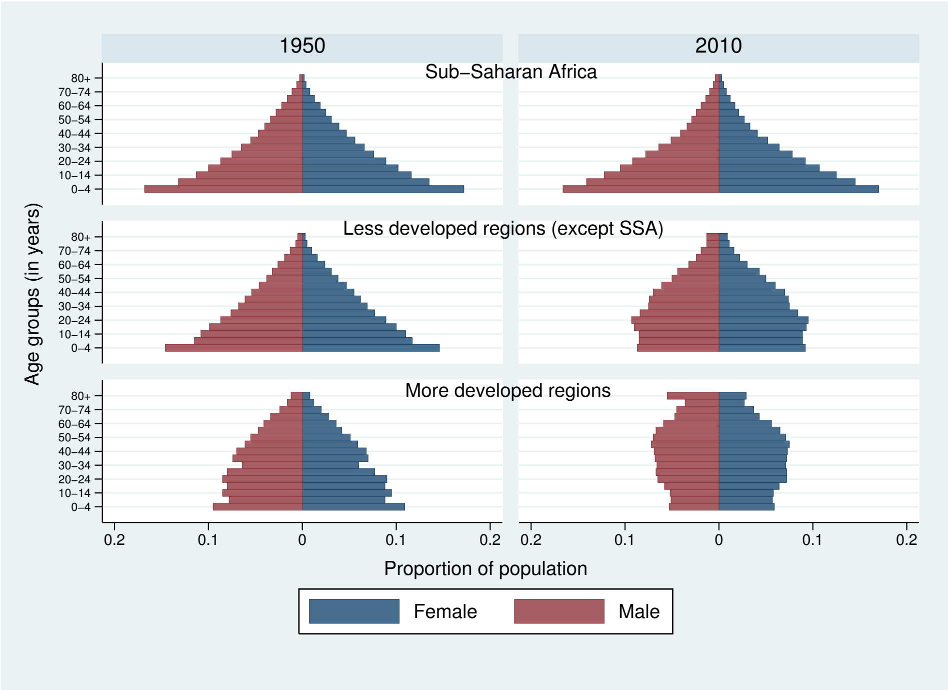

Figure 7: Age pyramids for the different regions of the world between 1950 and 2010

SSA is an exception to these trends among less developed regions. As shown in the figure, the age structure of the region has not significatively changed between 1950 and 2010 and remains similar to that of other less developed regions 50 years ago. SSA infant mortality rates remain very high indeed, at 77 per thousand in 2010. This results in a very low life expectancy as illustrated in Figure 8. The mortality risk is very high during early childhood and begins to decrease from five years of age. In general, Figure 8 shows an increase of life expectancy at all ages between 1970 and 2010, with a more significant improvement for newborns and children. Life expectancy at birth increased from 46 years in 1970 to 54 years in 2010, an annual average increase of 0.4%. SSA fertility rates also remain high and overcompensate for the mortality levels. Demographic growth in SSA is the highest in the world, at 2.5% per year in 2010. The population expanded fivefold over the 1950-2010 period, for an average annual growth rate of 2.6%. As a consequence, population in SSA is largely composed of young people leading to a high dependency ratio with respect to other less developed regions. More than half of the population in SSA is young (aged less than 24 years). Considering this particular demographic context, how does the assessment of social welfare differ when changes in population size and longevity are also considered in addition to changes in income distribution? The following sections provide some insights into this question.

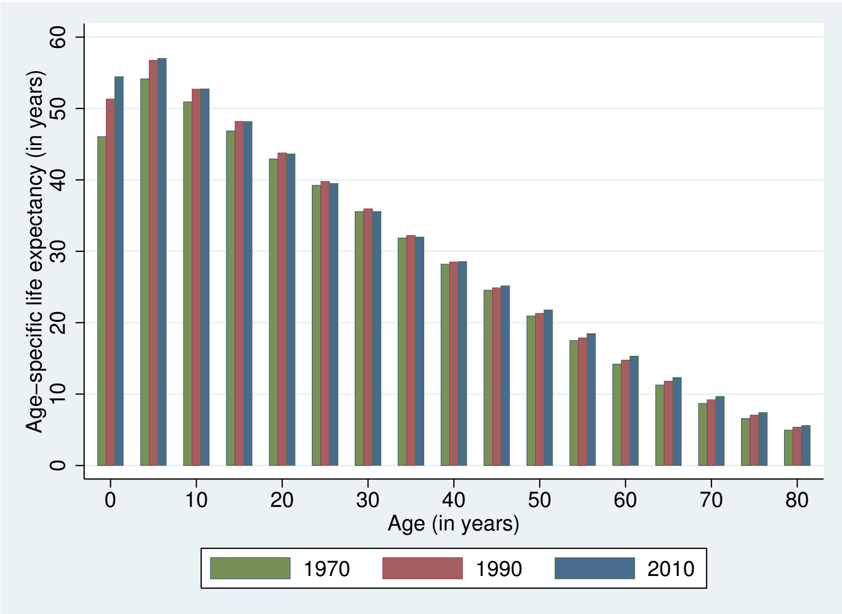

Figure 8: Age-specific life expectancy in SSA between 1970 and 2010

5 Welfare comparisons in a context of demographic changes: The case of Sub-Saharan Africa

In this section, we illustrate the different dominance criteria defined previously by performing welfare comparisons in a context of demographic change. As mentioned earlier, this amounts to comparing social states of the world defined by a utility matrix with two dimensions: individuals and time periods. Such matrices are defined for different regions of the world when performing regional comparisons and are defined for different time periods for temporal comparisons. We are particularly interested in assessing the temporal evolution of welfare in Sub-Saharan Africa during the last century by accounting for changes in incomes, longevity and population size. Thus, for some date t, we need the distribution of welfare over the course of the life of each Sub-Saharan African citizen alive at date t, including his past and future utilities. Incomes in each period are used as a proxy for utilities in those periods. Thus, the main challenge of the empirical application is to obtain the lifecycle distributions of income, which are not provided by household living standards surveys. In the next sub-sections we describe the methodology used to build the matrices of temporal income distributions and present some of the results.

5.1 Estimation of matrices of income distribution

The limited empirical literature on intertemporal welfare relies on household panel surveys, which provide the best data on temporal income distribution (Deaton and Paxton, 1994; Hoy et al., 2010; Maasoumi and Zandvakili, 1986, 1990; Rodgers and Rodgers, 1993; Salas and Rabadan, 1998). Unfortunately, the longest household panel datasets usually cover roughly thirty years and thus do not encompass entire lifetimes. We thus need an estimate of the lifecycle distribution of incomes in a given population using available household survey data. One approach is used in labour economics to estimate an individual's earning profile over the course of his life as a function of personal characteristics and his economic environment (see, for example, Heckman et al., 2003; Lemieux, 2006; Mincer, 1974). But these methods cannot be used to estimate income before entry into the job market, and also do not account for the changing income distribution within the population.

Our estimation technique relies on the three different types of data presented previously: *i*)the distribution of income among individuals for each year from 1820 to 2010, *ii*)population sizes, age structures and life expectancies by age in each year and *iii*)an income transition matrix from one year to another. For simplicity, we apply an estimated 2008/2009 decile transition matrix of Egypt for each year of the studied period. This decile income transition matrix is estimated from the Egyptian quintile income transition matrix provided by Marotta and Yemtsov (2010).

Table 1: Income transition matrix, Egypt, 2008-2009 (estimated)

	Decile income group 2009										
Decile income group 2008	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	Total
1	**0.43**	0.20	0.15	0.05	0.07	0.04	0.04	0.01	0.00	0.01	1.00
2	0.22	**0.37**	0.19	0.11	0.07	0.02	0.01	0.01	0.00	0.00	1.00
3	0.13	0.21	**0.22**	0.17	0.10	0.04	0.06	0.04	0.03	0.01	1.00
4	0.07	0.10	0.14	**0.20**	0.23	0.11	0.09	0.04	0.00	0.02	1.00
5	0.03	0.07	0.13	0.18	**0.20**	0.15	0.10	0.07	0.04	0.04	1.00
6	0.05	0.08	0.05	0.11	0.10	**0.15**	0.22	0.12	0.08	0.03	1.00
7	0.03	0.01	0.05	0.10	0.12	0.22	**0.25**	0.10	0.08	0.04	1.00
8	0.02	0.02	0.05	0.07	0.04	0.12	0.13	**0.23**	0.23	0.08	1.00
9	0.00	0.01	0.03	0.03	0.04	0.12	0.08	0.24	**0.35**	0.11	1.00
10	0.01	0.00	0.01	0.01	0.02	0.04	0.03	0.08	0.23	**0.55**	1.00

Source: Authors' estimates using Egypt transition matrix of Marotta and Yemtsov (2010).

Within this empirical framework, we estimated the income distribution matrices for the dates 1910, 1960 and 2010 using the procedure described below:

1. For each year from 1820 to 2070, we generate samples of annual income distributions, weighted by population size, using data from Bourguignon and Morrisson (2002) and

the growth rates of per capita income published by the World Bank (2013) for recent periods;

2. Using information on age structures and life expectancies in the population at their respective ages, we assign an age and a date of the death to each individual in each year of the analysis: 1910, 1960 and 2010;

3. For each individual in the sample, we generate prospective and retrospective income deciles using current income and the estimated decile transition matrix and its transposition, respectively;

4. Given an individual's decile at a given year, we draw annual incomes for the corresponding year from the samples of income generated in step 1.

5.2 Empirical results

5.2.1 Evolution of overall welfare in SSA over the last century

Using the estimation technique described previously, we constructed two matrices of lifecycle income distributions for the dates 1910 and 2010. We then rank the two matrices using the different intertertemporal social evaluation criteria discussed and developed previously. To do this, we apply the dominance criteria to the $OPEP$ income distribution vector of the matrix distribution. Thus, we first transform the matrices of income distributions into vectors of income distributions by concatenating lifecycle distributions of the different individuals. We then rank the $OPEP$ income distributions using the different social evaluation criteria.

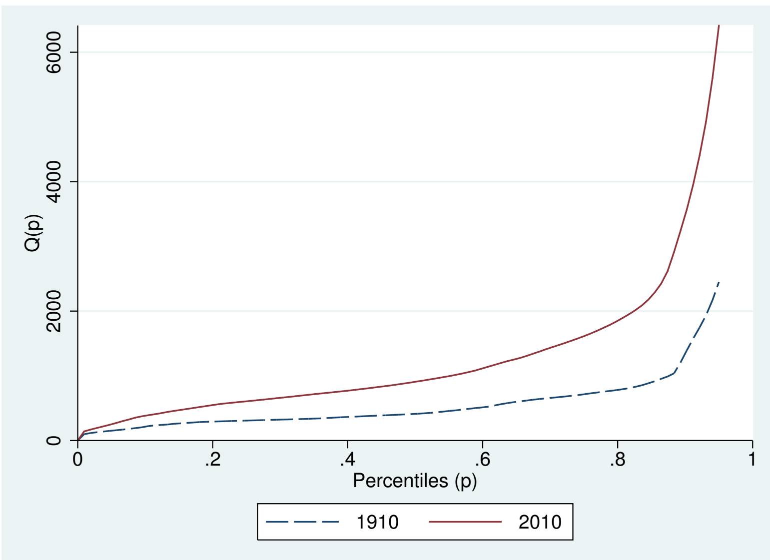

Figure 9: Intertemporal quantile curves dominance of 2010 versus 1910 for SSA using OPEP income distributions

Figure 9 presents the first-order dominance results using the intertemporal quantile curve ordering. It shows that even when changes in population size and longevity are taken into account, social welfare in SSA can be deemed to have increased significantly between 1910 and 2010. As expected, this first-order dominance leads to the second-order dominance as measured by the intertemporal generalized Lorenz dominance (Figure 16 in the Appendix). This means that despite the explosion of the population and a more limited improvement in longevity observed in SSA over the last century, 2010 strongly dominates 1910.

We then perform the welfare assessment using the multi-period critical-level population principle developed in Section 3. To do this, we first need to define the value of the multi-period critical-level α, which is an essential element in the analysis. Given the previously mentioned criticisms regarding the choice of α, here we investigate the dominance of 2010 versus 1910 for a set of critical-level values, using the multi-period critical-band utilitarian dominance criterion. The results indicate that social welfare in SSA can be shown to have increased significantly between 1910 and 2010 – i.e. the 2010 critical-level temporal generalized concentration curve is never beneath that of 1910 – for $\alpha = 0$ (Figure 17 in the Appendix). Indeed, this is true for all values of α up to \$180 (Figure 10). Using Corollary 1, this implies the dominance of 2010 over 1910 for all values of $\alpha \in [0, 180]$.

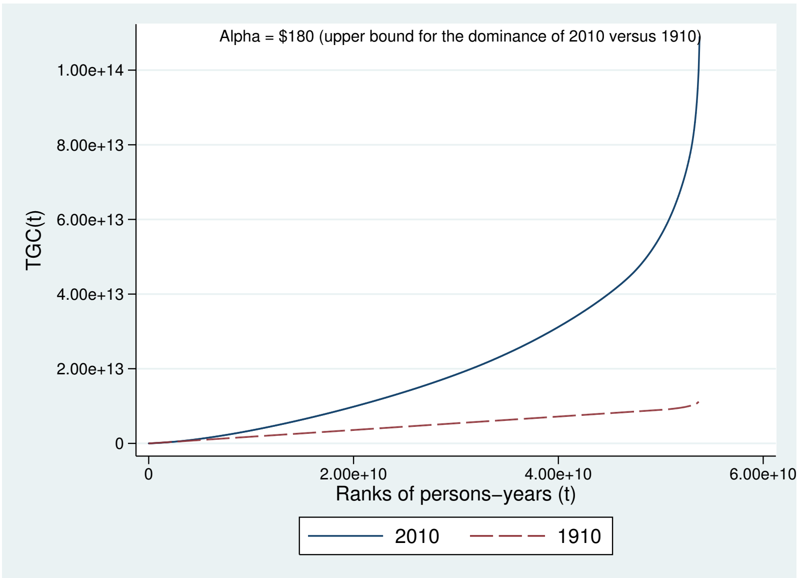

Figure 10: Multi-period CLGU dominance of 2010 versus 1910
for SSA using OPEP income distributions

From an ethical perspective, an upper bound for the critical level of \$180, or approximately half of the dollar-a-day poverty line, appears to be reasonable. Higher values for the critical level imply that the existence of individuals whose utilities in each period are below the poverty line is welfare-decreasing, even if the welfare of the existing population is unaffected. The estimates of the upper bound of the multi-period critical level (for dominance of 2010 versus 1910) may be viewed as the income threshold at which targeting populations with birth-control (or pro-natalist) policies increases (decreases) welfare. To get an idea of the magnitude of these dominance relationships, we compute numerical values and changes of social welfare from 1910 to 2010 for some specific functions.

5.2.2 Numerical illustration using cardinal measures of social welfare

Tables 2 and 3 provide numerical values and changes (in %) of social welfare in SSA over the last century for the main atemporal and intertemporal social evaluation functions. Results of the different value functions indicate the contribution of length of life and population size in social welfare. For a given date (1910, 1960 or 2010), the atemporal case computes values and changes in social welfare by considering the distribution of incomes in that year instead of considering the lifecycle distributions of incomes as done in the intertemporal case. Thus, the length of life and the lifecycle approach only come into play in the intertemporal case. We consider here the class of second-order inequality-averse value functions[3], which use a transformation of utilities $g(u) = log(u)$. Four types of social evaluation principles are considered: the Average Generalized Utilitarianism (AGU), the Classical Generalized Utiliatarianism (CGU), the Periodic Critical-Level Generalized Utilitarianism (P-CLGU) and the Lifetime Critical-Level Generalized Utilitarianism (L-CLGU). We set the periodic critical value $\alpha = \$180$, which is equal to the upper bound of critical for the dominance of 2010 over 1910 determined previously. The lifetime critical-level is simply the periodic critical-level times 60 years (a constant length of life arbitrarily fixed around the current value of life expectancy at birth in SSA).

Table 2: Values of social welfare in SSA from 1910 to 2010, case 1

	Atemporal			Intertemporal		
	1910	1960	2010	1910	1960	2010
AGU*	6.24	6.54	6.90	6.19	6.56	6.96
CGU	440.34	1204.82	4897.49	22820.49	81939.92	373597.89
P-CLGU	73.98	247.58	1210.63	3670.27	17068.46	94285.88
L-CLGU				81.52	380.90	1947.07

* All the values are in billions of log-dollars except that of the AGU function which are in log-dollars

Three main findings from this numerical illustration are worth noting. First, all the social evaluation functions confirm the continous improvement of welfare in SSA over the

[3] Results for the first-order value functions are provided in the Appendix.

period 1910–2010 as shown by the ordinal rankings presented previously. Second, the magnitudes of increase in social welfare are more important in the intertemporal case, highlighting the positive contribution of increased longevity in social welfare. Finally, the per capita approach, as illustrated by the AGU principle, concludes that social welfare has not significatively increased between 1910 and 2010 (only +12% over a century) compared to the other methods which show an important increase in social welfare (more than tenfold between 1910 and 2010). Whereas the per capita income-based approach reveals a limited improvement in social welfare in SSA between 1910 and 2010 due to the rise of inequality and an explosive population growth, the CGU and the CLGU principles show an increase in social welfare given the positive contribution of population growth and increased longevity. Note also that the P-CLGU and the L-CLGU approches lead to the same results in terms of relative improvement in social welfare. To confirm the contributions of population growth and increased longevity in welfare assessment, we cover next the impacts of changes in population size and life expectancy on the evolution of social welfare in SSA from 1910 to 2010 by simulating some hypothetical demographic patterns using the multi-period critical level approach.

Table 3: Changes in social welfare in SSA from 1910 to 2010, case 1

	Atemporal			Intertemporal		
	1910-1960	1960-2010	1910-2010	1910-1960	1960-2010	1910-2010
AGU	5%	6%	11%	6%	6%	12%
CGU	174%	306%	1012%	259%	356%	1537%
P-CLGU	235%	389%	1536%	365%	452%	2469%
L-CLGU				367%	411%	2288%

5.2.3 Effects of population size and life expectancy on social welfare

In order to isolate the net effect of changes in population size and life expectancy on social welfare, we consider some hypothetical scenarios and simulate the resulting social rankings in the case of SSA. The two main scenarios considered are: 1) the population remains constant while life expectancy and incomes vary as observed from 1910 to 2010 and 2) life expectancy remains constant while population size and incomes vary as observed from 1910 to 2010.

Figure 11 indicates that if the population had not expanded over this time frame (scenario 1), 2010 would dominate 1910 for all values of α. This result is consistent with a social ranking based solely on per capita income since critical-level utilitarianism is equivalent to average utilitarianism in this scenario. This implies that, in the present case,

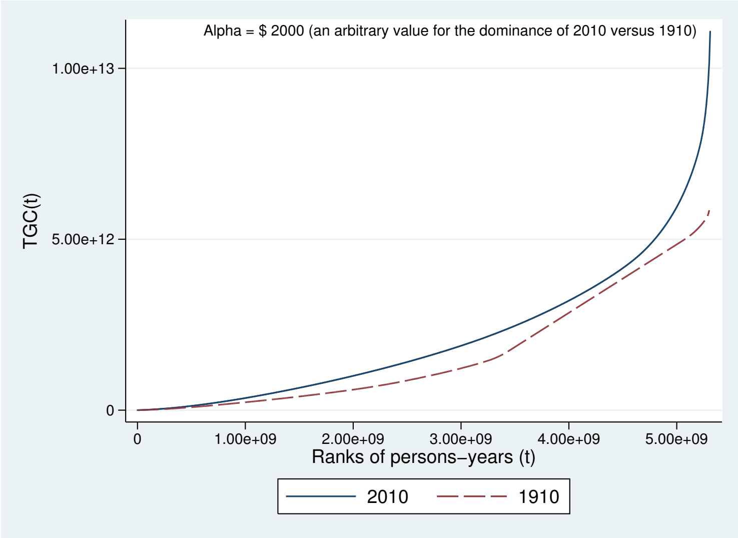

Figure 11: Multi-period CLGU Dominance of 2010 versus 1910
for SSA with constant population size (1910)

the demographic growth observed in SSA between 1910 and 2010 appears to have worsened the social welfare of the region if we use a critical-level value greater than \$180. This corroborates recent findings that population growth and economic development appear to be negatively related in poor countries (Weil and Wilde, 2009).

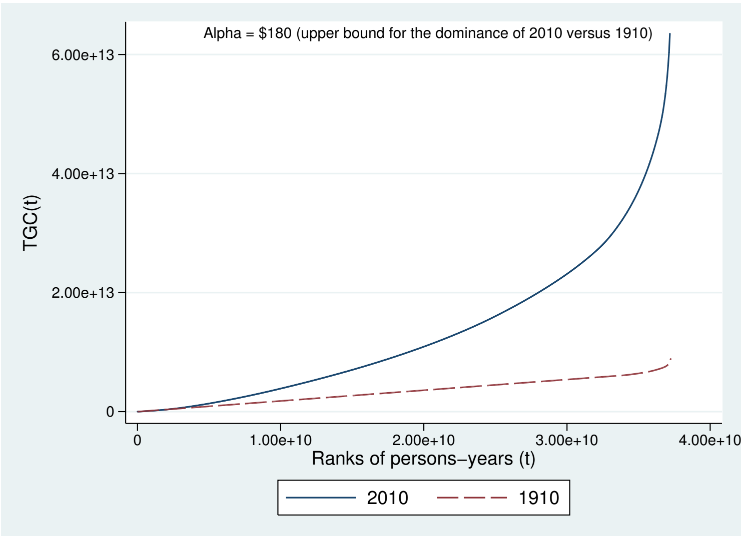

Figure 12: Multi-period CLGU dominance of 2010 versus 1910
for SSA with constant life expectancy 1910

Figure 12 shows that the social ranking results would have been the same even if life expectancies had not increased between 1910 and 2010 (scenario 2). This suggests that the effect of changes in population size largely dominates that of life expectancy according to the critical-level approach. Recall that the multi-period critical-level utilitarian function exhibits some degree of substitutability between population size and life expectancy. In this context, it appears as though the effect of life expectancy on social welfare is not significant due to the exponential increase of population size observed during the same period. This result to some extent clarifies recent findings by Cervellati and Sunde (2009), suggesting that life expectancy does not have a clear impact on economic development until the onset of the demographic transition, but this effect is positive afterwards.

6 Conclusion

In a context of demographic change, public policies are often evaluated by comparing social states of the World in which population size and longevity differ. This requires social evaluation principles to be set in an intertemporal framework that allows for trade-offs between standards of living, longevity and population size. Using the intertemporal social evaluation functions developed in the literature (Blackorby et al., 1995; Duclos and Housseini, 2013), this paper establishes a dominance criterion for welfare comparisons when populations differ in size and longevity and test it using historical data on population size, longevity and income distributions. More precisely, we build on dominance criteria for timeless social evaluation principles (Kakwani, 1984; Shorrocks, 1983; Trannoy and Weymark, 2009) and establish multi-period critical-level utilitarian dominance, defined by setting up a temporal version of the generalized concentration curve. We ultimately use this dominance criterion, as well as other intertemporal social evaluation principles in the literature, to assess the evolution of welfare in Sub-Saharan Africa between 1910 and 2010, jointly considering changes in and levels of life expectancy, population size and income distributions. This involves estimating the lifecycle income distribution of Sub-Saharan Africans in 1910 and 2010.

We contrast social ranking based solely on per capita income with the results using intertemporal social evaluation criteria such as the multi-period critical-level utilitarian dominance developed in the paper. Whereas the per capita income-based approach reveals a limited improvement in welfare in SSA between 1910 and 2010, the multi-period critical-level utilitarian approach shows an increase in social welfare if and only if the critical level of annual income – at which a life can be considered as social welfare-increasing – is less than or equal to \$180, i.e. roughly half the well-known dollar-a-day poverty line. We found that, when we use a critical value of greater than \$180, the demographic growth observed in SSA between 1910 and 2010 appears to have worsened social welfare for some intertemporal critical level social welfare functions. Our estimates of the upper bound of the critical value (for the dominance of 2010 over 1910) may be viewed as the income threshold at which targeting a population for birth-control policies unambiguously increases welfare. We also found that according to the critical-level approach, the effect of the life expectancy on social welfare in Sub-Saharan Africa is not significant given the exponential increase of population size observed during the same period. This result to some extent clarifies recent findings by Cervellati and Sunde (2009) that indicate that life expectancy does not have a clear effect on economic development until the onset of the demographic transition, while this impact is positive afterwards.

Appendix

A.1. Proof of Theorem 1

Proof. Consider $\alpha \in \mathbb{R}$ and $U, U' \in \mathcal{M}_u$.

From (12), $\widetilde{u} \succeq_{\alpha}^{TGC} \widetilde{u}' \Leftrightarrow TGC_{\widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')}} \geq TGC_{\widetilde{u}'_{\alpha,\overline{T}(\widetilde{u}',\widetilde{u})}}$.

For fixed populations with the same longevity, it follows from their definitions that temporal generalized Lorenz dominance coincides with multi-period critical-level α generalized concentration curve dominance for any value of $\alpha \in \mathbb{R}$, that is to say that in the case of populations with the same size and the same longevity,

$$U \succeq_{\alpha}^{PCL} U' \Leftrightarrow \widetilde{u} \succeq_{\alpha}^{TGC} \widetilde{u}'. \tag{19}$$

For variable populations with different life expectancies, we use the multi-period critical-level principle and build on the augmented temporal utility vectors to show the equivalence. If $n \neq n'$ and/or $T \neq T'$, we know that $U \sim \widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')}$ and $U' \sim \widetilde{u'}_{\alpha,\overline{T}(\widetilde{u'},\widetilde{u})}$. Thus,

$$U \succeq_{\alpha}^{PCL} U' \Leftrightarrow \widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')} \succeq_{\alpha}^{PCL} \widetilde{u'}_{\alpha,\overline{T}(\widetilde{u'},\widetilde{u})}. \tag{20}$$

Since $\widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')}$ and $\widetilde{u'}_{\alpha,\overline{T}(\widetilde{u'},\widetilde{u})}$ are vectors of the same dimension, it follows from the reasoning leading to (19) that:

$$\widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')} \succeq_{\alpha}^{PCL} \widetilde{u'}_{\alpha,\overline{T}(\widetilde{u'},\widetilde{u})} \Leftrightarrow \widetilde{u}_{\alpha,\overline{T}(\widetilde{u},\widetilde{u}')} \succeq_{\alpha}^{TGC} \widetilde{u'}_{\alpha,\overline{T}(\widetilde{u'},\widetilde{u})}. \tag{21}$$

Thus, (12), (20) and (21) imply that $U \succeq_{\alpha}^{PCL} U' \Leftrightarrow \widetilde{u} \succeq_{\alpha}^{TGC} \widetilde{u}'$. □

A.2. Proof of Corollary 1

Proof. It follows trivially from the definition of $\succeq_{[\underline{\alpha},\overline{\alpha}]}^{PCB}$ that 1 implies 2. We can now see that 2 implies 1.

Consider any $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, and $U, U' \in \mathcal{M}_u$. Thus, the utility vectors of their corresponding α one-person equivalent population (OPEP) are given by $\widetilde{U} \in \mathbb{R}^{\widetilde{T}}$ and $\widetilde{U}' \in \mathbb{R}^{\widetilde{T'}}$. Similar to the proof of proposition 3 by Trannoy and Weymark (2009), there are two cases to consider:

Case 1: $\widetilde{T} \geq \widetilde{T}'$, we have:

$$\begin{aligned} U \succeq^{PCL}_{\overline{\alpha}} U' &\Leftrightarrow W^{PCL}_{\overline{\alpha},g}(U) \geq W^{PCL}_{\overline{\alpha},g}(U') && (22)\\ &\Leftrightarrow W^{PCL}_{\overline{\alpha},g}(\widetilde{U}) \geq W^{PCL}_{\overline{\alpha},g}(\widetilde{U}') && (23)\\ &\Leftrightarrow \sum_{t=1}^{\widetilde{T}}[g(\widetilde{u}_t) - g(\overline{\alpha})] \geq \sum_{t=1}^{\widetilde{T}'}[g(\widetilde{u}'_t) - g(\overline{\alpha})] && (24)\\ &\Leftrightarrow \sum_{t=1}^{\widetilde{T}} g(\widetilde{u}_t) \geq \sum_{t=1}^{\widetilde{T}'} g(\widetilde{u}'_t) + (\widetilde{T} - \widetilde{T}')g(\overline{\alpha}) && (25) \end{aligned}$$

Because $(\widetilde{T} - \widetilde{T}') \geq 0$, $\alpha \leq \overline{\alpha}$, and the function g is increasing, it follows that:

$$\sum_{t=1}^{\widetilde{T}} g(\widetilde{u}_t) \geq \sum_{t=1}^{\widetilde{T}'} g(\widetilde{u}'_t) + (\widetilde{T} - \widetilde{T}')g(\alpha) \quad (26)$$

or equivalently,

$$\sum_{t=1}^{\widetilde{T}}[g(\widetilde{u}_t) - g(\alpha)] \geq \sum_{t=1}^{\widetilde{T}'}[g(\widetilde{u}'_t) - g(\alpha)] \Leftrightarrow U \succeq^{PCL}_{\alpha} U' \quad (27)$$

Case 2: $\widetilde{T}' > \widetilde{T}$, then 2 implies that:

$$\begin{aligned} U \succeq^{PCL}_{\underline{\alpha}} U' &\Leftrightarrow W^{PCL}_{\underline{\alpha},g}(U) \geq W^{PCL}_{\underline{\alpha},g}(U') && (28)\\ &\Leftrightarrow W^{PCL}_{\underline{\alpha},g}(\widetilde{U}) \geq W^{PCL}_{\underline{\alpha},g}(\widetilde{U}') && (29)\\ &\Leftrightarrow \sum_{t=1}^{\widetilde{T}}[g(\widetilde{u}_t) - g(\underline{\alpha})] \geq \sum_{t=1}^{\widetilde{T}'}[g(\widetilde{u}'_t) - g(\underline{\alpha})] && (30)\\ &\Leftrightarrow \sum_{t=1}^{\widetilde{T}} g(\widetilde{u}_t) + (\widetilde{T}' - \widetilde{T})g(\underline{\alpha}) \geq \sum_{t=1}^{\widetilde{T}'} g(\widetilde{u}'_t) && (31) \end{aligned}$$

Since $(\widetilde{T}' - \widetilde{T}) > 0$, $\alpha \geq \underline{\alpha}$, and the function g is increasing, it follows that:

$$\sum_{t=1}^{\widetilde{T}} g(\widetilde{u}_t) + (\widetilde{T}' - \widetilde{T})g(\alpha) \geq \sum_{t=1}^{\widetilde{T}'} g(\widetilde{u}'_t) \quad (32)$$

or equivalently,

$$\sum_{t=1}^{\widetilde{T}}[g(\widetilde{u}_t) - g(\alpha)] \geq \sum_{t=1}^{\widetilde{T}'}[g(\widetilde{u}'_t) - g(\alpha)] \Leftrightarrow U \succeq^{PCL}_{\alpha} U' \quad (33)$$

Thus, in both cases, for any $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, we have shown that:

$U \succeq^{PCB}_{[\underline{\alpha},\overline{\alpha}]} U' \Rightarrow U \succeq^{PCL}_{\alpha} U'$. That is, 2 $\Rightarrow$ 1.

Finally, the equivalence of 3 with both 1 and 2 follows directly from theorem 1. □

A.3. Atemporal social ranking in SSA over the last century

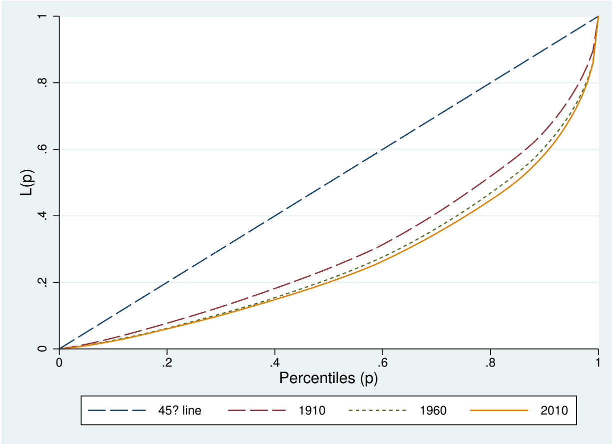

Figure 13: Lorenz curves of income distributions in SSA over the last century

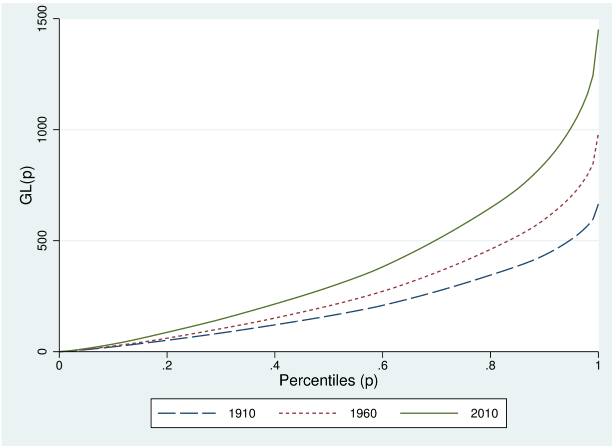

Figure 14: Generalized Lorenz curves of income distributions in SSA over the last century

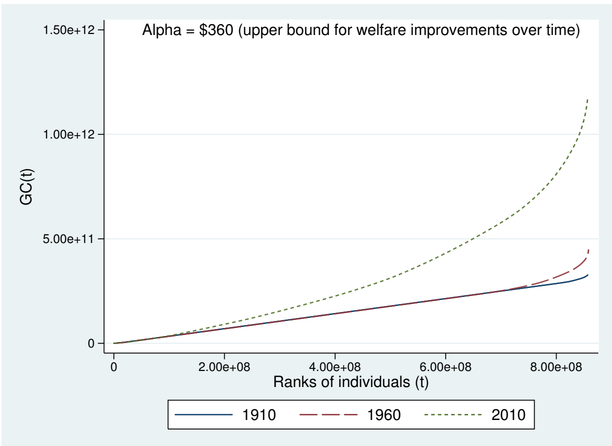

Figure 15: Critical-level concentration curves of income distributions in SSA

A.4. Intertemporal social ranking in SSA over the last century

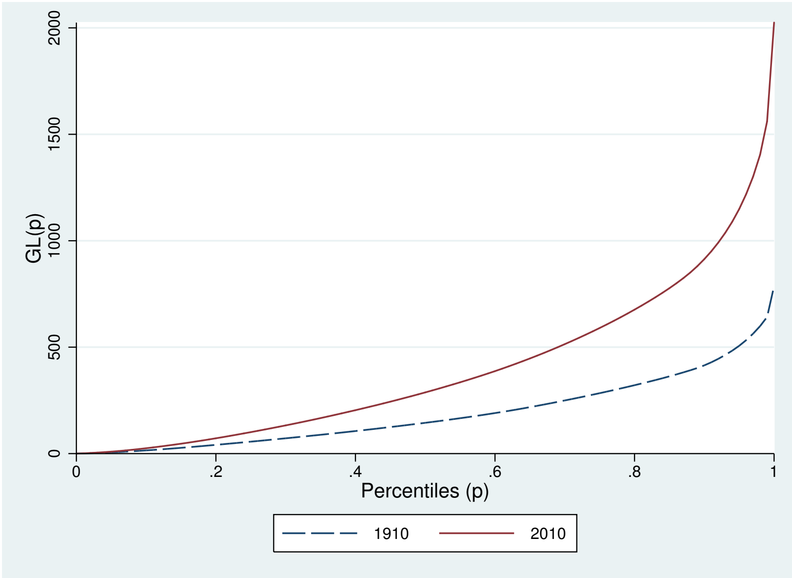

Figure 16: Intertemporal Generalized Lorenz dominance of 2010 versus 1910 for SSA

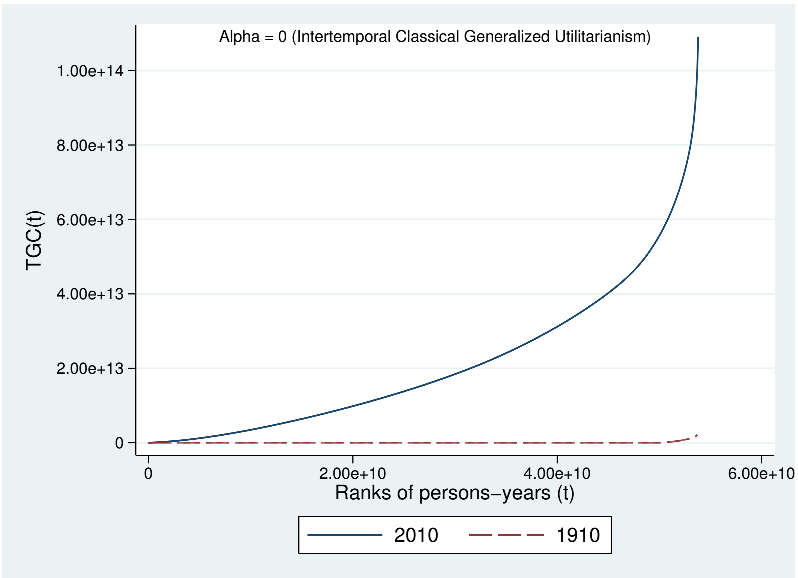

Figure 17: Multi-period CLGU Dominance of 2010 versus 1910 for SSA with α equal to zero

A.5. Numerical values and changes of social welfare in SSA using social evaluation functions with $g(u) = u$

Table 4: Values of social welfare in SSA from 1910 to 2010, case 2

	Atemporal			Intertemporal		
	1910	1960	2010	1910	1960	2010
AU*	673.91	994.43	1480.55	775.96	1316.97	2053.81
CU	47.54	183.31	1051.15	2902.03	16920.05	108914.99
P-CLU	34.84	150.13	923.35	2238.24	14671.45	99233.39
L-CLU				1986.19	14493.29	99644.79

* All the values are in billions of dollars except that of the AU function which are in dollars

Table 5: Changes in social welfare in SSA from 1910 to 2010, case 2

	Atemporal			Intertemporal		
	1910-1960	1960-2010	1910-2010	1910-1960	1960-2010	1910-2010
AU	48%	49%	120%	70%	56%	165%
CU	286%	473%	2111%	483%	544%	3653%
P-CLU	331%	515%	2550%	555%	576%	4334%
L-CLU				630%	588%	4917%

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