Key sectors in economic development: a perspective from input-output linkages and cross-sector misallocation

Julio Leal
Banco de Mexico

May 13, 2015

Version 1.0

Abstract

For a typical developing country, this paper shows that once inter-sectoral linkages are taken into account, closing the productivity gap in a number of services gives bigger gains in aggregate productivity than closing it in agriculture or in manufacturing, despite their larger gaps. This is performed in the context of an input-output economy and general equilibrium. Also, the importance of sector-specific distortions that produce cross-sector misallocation is addressed. I compute the effect of the removal of these distortions on aggregate productivity using the input-output model and find that this could increase productivity up to 67%, depending on whether the rents from distortions stay in the economy or not.

1 Introduction

Which sectors make poor countries so unproductive? One common idea is that there exist large distortions in a few key sectors that explain the bulk of the gap in aggregate productivity between rich and poor countries. The development literature have traditionally emphasized problems in agriculture and/or manufacturing\(^1\) (recent examples are Restuccia et al. (2008), Herrendorf and Teixeira (2005), and Buera et al. (2009)). In contrast, a recent branch of this literature emphasizes distortions prevalent in services, such as those associated with the presence of informality. For example, Prado (2011), and D’Erasmo and Moscoso Boedo (2012) argue that informality is associated with resource misallocation and other distortions. Thus, which sectors are the most important ones for explaining the differences in aggregate productivity across countries, is still an open question.

The role of resource misallocation across plants has recently been emphasized in the development literature as an explanation for the large differences in productivity across countries (e.g., Restuccia and Rogerson (2008) and Hsieh and Klenow (2007)). In the same spirit, if sector-specific distortions are in place, cross-sector misallocation occurs. These distortions disrupt the equalization of marginal products across sectors undermining aggregate productivity\(^2\). What is the quantitative importance of this type of misallocation on aggregate productivity?

---

1Restuccia et al. (2008) blames the barriers to the use of intermediate inputs in Agriculture; Herrendorf and Teixeira (2005) emphasize barriers to international trade that directly affect industries that produce tradables; and Buera et al. (2009) argue that the problem is financial frictions that affect manufactures more than services.

2Of course, sector-specific distortions might simply be the result of firm-level distortions that differ across sectors.
I make two main arguments regarding the questions at hand. First, I argue that policies that affect the productivity of *highly interconnected sectors* are important determinants for aggregate productivity. Consider the following series of events. If the productivity of refined petroleum is low, this affects gasoline production, which in turn affects transportation, which affects trade, which affects back to the production of refined petroleum products, and so on. Thus, I argue that it matters not only which sectors have the largest productivity gap with respect to the leader, but also the “degree of influence” of each sector. Furthermore, this degree of influence is determined by the heterogeneity in input-output relationships across sectors. Thus, it is natural to think on a *key sector* as one with both, a large productivity gap and a large degree of influence. The first goal of this paper is to identify which are the key sectors for a typical developing country.

The second argument in the paper is regarding sector-specific distortions faced by firms in developing countries that are not directly linked to low productivity at the *industry level*, but that could be a source of cross-sector misallocation, and thus, have an impact on aggregate productivity. An example of these distortions are polices and/or market structures that introduce a wedge between marginal revenue and marginal cost (such as the presence of imperfect competition) in specific industries which might not necessarily translate into low productivity at the industry level. However, the presence of this wedge could still produce resource misallocation and distort other margins of the economy, affecting measures of aggregate productivity.

To achieve these goals, I use a multi-sector model with inter-sectoral linkages based on Long Jr and Plosser (1987), Acemoglu et al. (2012), and Jones (2011b, b). In the model, there are $N$ sectors (or industries) that produce different goods. The output of each sector can be used either as consumption or as an intermediate input in the production of the other sectors. This introduces a link between the performance of an individual sector and the performance of the rest. Thus, when the performance of a sector is improved, say by increasing its productivity or by reducing its distortions, the final impact on the aggregate economy will be determined by the way this sector interacts with other sectors through input-output relationships.

In the spirit of Chari et al. (2007) I analyze two types of distortions that misallocate resources across sectors: 1) distortions that show up as a wedge between marginal revenue and marginal cost; and 2) distortions that introduce a wedge between the marginal product of labor and the marginal cost of labor. I measure these distortions at the industry level. The first distortion enters in the firm’s profit maximization problem as an output tax, and can also be interpreted as a markup that rises price over marginal cost. This distortion affects the share of value added over gross output of individual industries, which is a feature of the data that I observe in Mexico (see section 2). In particular, I show that, for the majority of the Mexican sectors this ratio is high, implying that the use of intermediate inputs is depressed with respect to the US. I call this distortion “the markup wedge”, for simplicity.

The second distortion, enters in the firm’s problem isomorphic to a payroll tax, and it captures policies that shift resources away from workers while increasing labor costs to firms; affecting the labor income share at the industry level. Gollin (2002) argued that the labor income share in developing countries is low due to measurement problems. In section 2, I show that even after performing Gollin’s measurement correction, the labor income share is still low for the majority of the Mexican sectors. I call this distortion “the labor wedge”.

The main contributions of the paper are as follow. First, I introduce the analysis of economic networks in the development literature, addressing the question of which countries make poor countries so unproductive from the perspective of inter-sectoral linkages, and pointing out to the need of recognizing input-output relationships to correctly identify “key sectors”. Second, I provide a quantitative assessment of the importance of distortions that produce cross-sector misallocation for a typical developing country, identifying the main economic channels through which these distortions affect productivity and output.

---

3See Acemoglu et al. (2012).
I use a calibration strategy that avoids the computation of productivity levels and instead focuses on productivity gaps. The strategy is as follows. I assume a Cobb-Douglas specification for the gross output production function in each sector, and calibrate the model for both, the US and Mexico. I have in total $N+5$ parameters to calibrate in each industry ($N=33$). In accordance with the development literature, I take the US as a relatively undistorted economy. Using the US as a reference point, I measure the productivity gap, and the distortions in Mexico. I use Mexico because given its strong economic relationship with the US, I expect small technological differences. Nonetheless, I keep to a minimum the number of parameters in the model that are assumed to have a common value in the two countries. I make two assumptions regarding the value of the parameters in the production function: 1) I assume that the parameter that controls the labor share in each industry ($\alpha_i$) is the same in the US and Mexico; and 2) I assume that the parameter that controls the share of value added in gross output is also common in both countries. The first assumption is standard in the development literature, and the second one is well supported by the data. Other than that, the remaining parameters are country specific, and are calibrated by matching moments in the data of each country, respectively. 

I use the calibrated parameters to compute the “vector of influence” implied by the model and to provide a quantitative assessment of the effect on aggregate GDP per worker of two counter-factual exercises: 1) closing sectoral productivity gaps; and 2) eliminating sectoral wedges.

The results are as follows. First, in line with previous literature, I show that Mexico’s productivity gap is larger in manufactures. However, in contrast to previous literature, I show that once interconnections are taken into account, closing the productivity gap in services, would give the biggest gains in GDP per worker. Accordingly, most of the key sectors are in services. To illustrate the mechanics behind this result, take two typical industries in manufacturing and services: Textile and Textile Products (sector 4), and Wholesale Trade (sector 20), respectively. The industry of Textiles in the US is 8 times more productive than the corresponding one in Mexico, while Wholesale Trade is only 3 times more productive. However, Trade is not only a much bigger sector than Textiles, it is also one of the most interconnected sector in the economy: the degree of influence of Trade is 5 times bigger than the degree of influence of Textiles. Therefore, closing the productivity gap in Trade gives much bigger gains in GDP per worker than closing it in Textiles (15% vs. 4% gains), despite the fact that the productivity gap is higher in Textiles.

One important feature of the model is that the equilibrium labor allocation across sectors is invariant to changes in productivity. This is a feature that makes cross-plant misallocation different than cross-sector misallocation. While in standard models of heterogeneous firms the allocation of resources is largely determined by relative productivity across firms; in standard multi-sector models, such as the one used in this paper, the allocation of resources across sectors is largely determined by the vector of influence, which, in turn, is affected by the specification of demand and by the nature of the inter-sectoral network.

To assess the importance of resource misallocation across sectors, I perform a counter-factual exercise that consists on eliminating the industrial wedges. As mentioned above, I assume no distortions in the US and compute the implied distortions in Mexico. In general, I find that marginal revenue tends to be above marginal cost for the majority of the sectors in Mexico. The unconditional average of industrial markup wedges is 1.3, while if we condition to industries with markup wedges above 1, the average is 1.6.

It becomes relevant to distinguish between two cases regarding the distribution of rents from distortions: when the rents are given back to the household as lump-sum transfers (case 1); as opposed to when the rents are lost and taken out of the economy (case 2). In the first case, the presence of wedges creates

---

4For the country-specific parameters, I use data contained in the input-output tables, such as the value of gross output, labor compensations, and the purchases of domestic and imported intermediate inputs by industry. Finally I also use data on relative prices of gross output in international dollars by sector to compute the sectoral labor productivity gaps.

5It is emphasized that this wedge is measured relative to the US. Thus, a markup smaller than 1 only implies that the distortion is lower in Mexico relative to the one prevalent in the US.
resource misallocation of labor across sectors, however, eliminating a markup does not necessarily increases GDP. This depends on whether the elimination of the markup brings the whole set of markup wedges closer to each other (i.e., whether dispersion is reduced, or not).

There are other margins in the economy that are also affected in case 1 when the markup wedge is reduced. I identify three: 1) an effect on the supply of the good; 2) an effect on the allocation of labor (just described); and 3) an effect on the allocation of output between final and intermediate uses. The first one is intuitive as the markup enters in the profit maximization problem of the firm like an output tax, which, when reduced, it increases marginal revenue. The third effect is present due to the negative income effect that occurs when reducing the rents associated with the distortion, this reduces aggregate demand and final consumption. As a result of these positive and negative forces, the total effect of eliminating all markups simultaneously is small.

The same amplification effect that is present when productivity gaps are closed, is also present when markups are eliminated. Consider the following example. Two sectors with large markups are Education and Real Estate. Note, however, that Real Estate has a large degree of influence, while Education does not. Thus, the direct effect on the supply of \( i \) will be big for the case of Real Estate, while it won’t be as big for the case of Education. In contrast, the negative income effect will be high in both cases (though, it will differ, in general). As a result, the effect of eliminating the markup wedge in education is negative, while the effect is positive for the case of Real Estate.

In case 2, when the transfers are not given back to the household, the markup wedges are isomorphic to productivity. The wedges do not create resource misallocation in this case, but these can have a sizable effect on aggregate output. The reason for this is that, just like a decrease in productivity, a markup wedge reduces the amount of output per unit of input, affecting GDP and aggregate productivity. As a consequence, the effect of eliminating markups is much bigger than in case 1: when all markup wedges are eliminated simultaneously, aggregate productivity increases 67.7%. This large effect is also explained by the existence of a “multiplier effect” that occurs through the input-output network: a 1% decrease in the markup, increases aggregate output in more than 1%. Intuitively, if we reduce the markup in Trade, this has an impact not only in the production of Trade, but in the production of all the sectors that use Trade as an intermediate input. In turn, the sectors that use the sectors that Trade uses as intermediate inputs as intermediate inputs are also benefited. Since there are not countervailing negative forces such in case 1, the total effect is large.

The contrasting effects on aggregate output between cases 1 and 2 is informative about the economic channel through which labor misallocation operates in the model. In particular, notice that the misallocation of labor is present in case 1 due to the extra income effect that transfers entail. When we reduce the distortion of sector \( i (\psi_i) \) the rents associated with that distortion are also reduced, and, as a result, there is less income to consume, overall. This, in turn, translates into less labor being allocated to every sector. However, the reduction of \( \psi_i \) increases the marginal revenue product of labor in sector \( i \), which mitigates the negative income effect on that sector. These forces reallocate labor into sector \( i \), and away from every other sector.

Finally, for the case of the labor wedge, I find that an overwhelming majority of the sectors show a positive wedge that increases the cost of labor to the firms. This wedge significantly reduces labor compensation as a fraction of value added. We find that on average, the marginal productivity of labor is 42% above marginal cost. Conditional on having a positive wedge, this number increases to 68%. An important fraction of the difference in the labor income share between México and the US is explained by the presence of the markup wedge, the rest of course is explained by the labor wedge. This implies that policies that tend to decrease competition affect the labor income share, as well as policies that divert resources from workers increase the cost of labor to the firms.

**Related literature.** A long tradition of studies argues that the productivity gap in poor countries manufacturing is higher than in services (e.g., Balassa (1964); Samuelson (1964); and more recently,
Buera et al. (2009), and Herrendorf and Valentinyi (2012). This is also true in the recent data from Inklaar and Timmer (2013), and in Herrendorf and Valentinyi (2012). This literature did not take into account the role of inter-sectoral linkages and cross-sector misallocation to assess which sectors are key for development.

A large literature intends to explain the sources of cross-country income differences. My paper is related to that literature and specially to a small subset studying the role of intermediate inputs in productivity. In particular: Moro (2011) and Jones (2011a, a). My paper is also related to the literature on resource misallocation across plants (e.g., Restuccia and Rogerson (2008) and Hsieh and Klenow (2007)). There is growing interest on extending the study of resource misallocation beyond the dimension of plants. Jones (2011b, b) argues that misallocation might be enhanced in input-output economies. I show that misallocation in multi-sector models is different than misallocation in heterogeneous firms models. In particular, I show that labor allocation is invariant to productivity changes in multi-sector models, while in heterogeneous firm models the allocation of labor is highly determined by relative productivity. In multi-sector models, the allocation obeys the structure of the demand side and the specification of the sectoral network. In contrast to Jones (2011b, b), I show that, what is crucial for aggregate productivity is whether the rents associated from distortions stay in the economy or are taken away.

The paper is also related to the literature of economic networks such as in Acemoglu et al. (2012) and Acemoglu et al. (2015). This literature has focused on the role of networks in business cycles. This paper is an application of the concept of “degree of influence” coined by Acemoglu et al. (2012) to the literature of economic development.

The literature that studies the low labor income share in developing countries is also related. For example, Ayala and Chapa (2014) argue that this share is low in Mexico even after correcting for the measurement issues addressed by Gollin (2002). It is also related to the literature studying a generalized recent decline of the labor share across countries, such as in Karabarbounis and Neiman (2014). In this paper, I argue that the low labor income share in Mexico is explained by the presence of the markup wedge, which in turn, might be related to lack of competition in product markets.

The organization of the paper is as follows. Section 2 presents relevant facts, section 3 presents the model and discusses the effect of distortions, section 4 presents the calibration strategy, section 5 the results, and section 6 concludes.

## 2 Facts

In this section, I present several facts that are relevant for the question at hand. First, as documented elsewhere, I show that the productivity gap in developing countries is larger in manufactures. Second, I show that there is substantial heterogeneity in the degree of interconnections across sectors as well as in their final consumption shares. These two features are important to determine the “degree of influence” of each sector. The concept of “degree of influence” is taken from Acemoglu et al. (2012) and it captures the idea that the stronger the inter-connections of a given sector, and the higher its final consumption share, the more “influential” a sector will be in the aggregate economy. The variability on this degree of influence across sectors motivates the main argument of the paper: mainly, that in order to determine how important the performance of a sector is for aggregate productivity, it is not sufficient to only look at its productivity gap with respect to the leader; instead, one has to look at both, the productivity gap, and the degree of influence, to assess such effect. The interaction between these two characteristics will combine to produce the final effect of closing the productivity gap of a given sector on aggregate productivity.

---

6Acemoglu et al. (2012) did not consider variation in consumption shares across sectors. However, once this variation is allowed in the model it turns out to be important for the vector of influence.
In addition to the productivity gap, I study two more sources of distortions in Mexican firms that could potentially lead to low aggregate productivity as well. In this section, I present two facts that motivate the introduction of “wedges” in the model. First, I show that the shares of value added in gross output at the industry level in Mexico are closely correlated with the shares in the US. That is, if one finds a US sector with a relatively high value added share in gross output, the corresponding sector in Mexico will also have a relatively high value added share in gross output. Nonetheless, the data indicates that these shares differ across the two countries, in particular, these tend to be bigger in Mexico for the majority of the sectors. Conversely, the data indicates that the Mexican shares of intermediate inputs in gross output (including imported imports) are below the corresponding US shares. This feature of reality will be later interpreted as the presence of “wedges” that distort the optimal decisions of firms.

Second, I present data on labor income shares in value added across the Mexican sectors. In general, the data shows that these shares, as well as the aggregate share, are low. The low labor share in Mexico is not explained by the arguments in Gollin (2002). I perform an exercise where I correct for the problems emphasized by this author at the aggregate level, that is, I measure the labor share taking into account the income that is not properly divided (between capital and labor income) and show that this does not significantly increases the labor share in Mexico. I will interpret these low labor shares as other kind of “wedges” that distort firm’s optimal conditions.

\section{Productivity gaps in manufacturing and services}

Here, I document that the productivity gap is larger in manufacturing relative to that in services. I use data from Inklaar and Timmer (2013) who compute cross-country relative prices at the industry level using data on prices of final goods. They report their estimates for the productivity of services and manufactures for a large number of developed and developing countries (for more details, see Inklaar and Timmer, 2013). We plot their estimates of the relative productivity of services vs. GDP per hour worked in Figure 1. As the figure shows, the poorer the country, the larger the relative productivity of services with respect to manufactures. This implies that the productivity gap in poor countries is larger in manufactures. For Mexico, the relative productivity of services is below the fitted line, but still above the value of most developed countries.

A second piece of evidence is the one found in Herrendorf and Valentinyi (2012), where the authors compute Total Factor Productivity (TFP) for three main aggregates: GDP, services and goods. They report the ratio of TFP in the US to TFP in Latin America (LA). The data from Herrendorf and Valentinyi (2012) is presented in Table 1. The table tells a similar story as the one in Figure 1. Mainly, that the productivity gap is bigger in manufactures. The productivity in the sectors that
Table 1: Relative TFP US vs Latin America for the aggregate, services and goods.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>$TTP^{US}/TTP^{LA}$</td>
<td>2.30</td>
</tr>
<tr>
<td>Services</td>
<td>$TTP^{SUS}/TTP^{SAL}$</td>
<td>1.86</td>
</tr>
<tr>
<td>Goods</td>
<td>$TTP^{GU}/TTP^{GAL}$</td>
<td>3.58</td>
</tr>
</tbody>
</table>

Source: Herrendorf and Valentin (2012).

Figure 2: Relative gross output labor productivity (MEX/US).

produce US goods is 3.58 times the corresponding productivity in Latin America; in contrast, this number is only 1.86 in the case of services.

A third and final piece of evidence is the data on gross output labor productivity that can be constructed using Inklaar and Timmer’s PPP estimates at the industrial level and data on gross output and hours from the World input-output database (WIOD) which is constructed by Timmer et al. (2012). We present such measures in Figure 2 for the Mexican sectors. The main message is similar to the one implied in the previous figure and table: the gaps tend to be larger in manufacture sectors. The average relative sectoral productivity is 0.30, which implies that the gap is $2.33 \left( = \frac{1 - 0.3}{0.3} \right)$. Note that the figure includes a label on top of the bars that indicate whether the sector belongs to services (label=1), or otherwise (label=0). While 58% of the sectors that have a lower than average gap are services, only 38% of the sectors with a larger than average gap are services. Put it differently, the majority of the sectors with large productivity gaps are manufactures.
2.2 Sectoral interconnections

Mexican sectors exhibit considerable heterogeneity in the interconnections as measured using the information in the input-output tables. Figure 3 shows a network map for the Mexican economy. Each circle is a sector. The area of the circle is determined by the final consumption share of the sector (i.e., a measure of the size of the sector). A string between two circles indicates that the economic transactions between them are significant (i.e., above some threshold). The more centric is a sector, the more interconnections the sector has. The figure shows that there is great heterogeneity across sectors in terms of, not only relative size, but also the number of inter-connections. Intuitively, the larger the consumption share, and the stronger its inter-connections, the more important role the sector will play in the economy.

2.3 Input shares as wedges

Previous literature has used variation in input shares either in gross output or in value-added across time and sectors to identify distortions on the optimal behavior of firms. When the production function is Cobb-Douglas and the firm operates under perfect competition, the equalization of the marginal product to the marginal cost of the inputs implies that the input shares are constant and equal to the coefficients in the production function. As a result, a discrepancy between input shares and the value of these coefficients, might be indicative of the presence of distortions. As explained by Cole and Ohanian (2013), deviations from perfect competition in product markets break the equality between the marginal product of inputs and the price of those inputs (the marginal cost). The reason is that under imperfect competition firms equate the marginal revenue product to the marginal cost, and not the marginal product, and thus, it depresses the quantity of inputs hired by the firm. One early contribution using the same basic idea is Hall (1988) who uses the ratio of labor compensation to total revenue to study the relation between price and marginal cost in US industries. More recently, there is the article of De Loecker (2011) who uses a similar property of firm’s maximization to identify markups in specific exporting industries. In general, variation in input shares can occur for several reasons, a simple one being the existence of taxes. Taxes distort the equalization of marginal products and marginal costs because part of the marginal product has to be put aside by the firm in order to comply with tax laws. In general, any regulation, pecuniary or not, that rises the cost of inputs to the firms will create variation in input shares. Similarly, any regulation that affects marginal revenue, will also create variation in input shares.

I concentrate on two kinds of shares: the value-added share in gross output, and the labor share in value added. The value-added share in gross output is the complement of the intermediate inputs share in gross output. The focus in these two shares is because their measurement is relatively more accurate than other inputs in production, such as capital.

2.3.1 Intermediate inputs share

Next, I show that the shares of value-added in gross output (the complement of the intermediate inputs shares) across Mexican industries have a strong correlation with the corresponding shares in the US. Figure 4 is a scatter plot of the US shares vs. the Mexican shares. The figure shows that if a share is relatively high for an specific industry in the US, then one could expect that the corresponding Mexican industry will also have a relatively high share.

Figure 5 shows the same plot but adding a 45 degree line. This figure indicates that despite the close correlation between Mexican and US shares, the Mexican industries tend to have a larger value added-share on gross output relative to the US industries. Alternatively, the data shows that the intermediate inputs shares in Mexico are depressed relative to the US ones. In the model, I will rationalize these
Figure 3: Network map of Mexican Sectors.

Source: Author’s calculation.
2.3.2 Labor share

The labor income share is low in México, as in many developing countries. It is commonly believed that this is due to the measurement arguments emphasized by Gollin (2002). The main argument made by Gollin is that in developing countries there is a substantial fraction of labor income that is recorded as non-labor income in national accounts. The main reason for this is the large presence of self-employment and unpaid family workers in developing countries.

Figure 6 presents a scatter plot of the sectoral labor shares for Mexico and the United States calculated using WIOD data. The WIOD makes a correction of labor compensations in developing countries to take into account the large presence of self-employment (see Timmer et al., 2012 for details), however it does not take into account the presence of unpaid family workers. For this reason, the WIOD data on labor compensation can still contain some downward bias, though smaller than a naive calculation that does not take the Gollin’s critique into account. The Figure shows that labor shares are positively correlated between Mexico and the United States, however, Mexico consistently exhibits lower labor shares.
Figure 6: Labor income share in value-added

Source: Author’s calculation using data from WIOD.

Table 2: Aggregate labor share: Naïve vs. corrected calculation

<table>
<thead>
<tr>
<th></th>
<th>Labor share (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico “naïve”</td>
<td>0.28</td>
</tr>
<tr>
<td>Mexico “corrected”</td>
<td>0.42</td>
</tr>
<tr>
<td>US</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Source: Author’s own calculation. The “naïve” calculation refers to the exercise of taking the ratio of labor compensations to GDP straight from National Accounts. The “corrected” calculation refers to the exercise described in Conesa et al., 2007.

In Table 2, I present an exercise to correct for the measurement problems emphasized by Gollin following the methodology proposed by Conesa et al., 2007. Due to the lack of information by sector, this exercise is performed using aggregate data. The methodology departs from the observation that the concept of “labor compensations” in National Accounts unambiguously corresponds to labor income. Thus, the idea is to identify the fraction of GDP that includes this concept and its corresponding capital income. Since ambiguous income is recorded as Net Mixed Income from the household sector, this is subtracted from GDP (together with Net indirect taxes), and then the ratio of “labor compensations” to this “adjusted” GDP is obtained. The table shows, that even performing this correction the Mexican labor income share remains well below to US share.

3 Model

The model here is a version of the one found in Long Jr and Plosser (1987), which was also recently used by Jones (2011b) and Acemoglu et al. (2012). Consider an economy with $N$ sectors. The supply of labor ($H$) is exogenous and each sector uses labor and commodities from all other sectors (including its own) to produce. We assume that the production function of a representative firm in sector $i$ is represented by the following Cobb-Douglas technology:

$$Q_i = A_i (H_i)^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i},$$
where $x_{ij}$ represents the intermediate demand that industry $i$ makes from industry $j$ and $M_i$ is the quantity of a foreign intermediate good imported by sector $i$. $A_i$ and $H_i$ represent an exogenous productivity term, and labor used in sector $i$, respectively. Also, we define $\sigma_i = \sum_{j=1}^{N} \sigma_{ij}$. Notice that this production function exhibits Decreasing Returns to Scale (DRS), an assumption that is taken without loss of generality. DRS has the advantage that it allows for a clear interpretation of the wedges, in particular, using this specification, makes it straightforward to relate the industrial labor share with the coefficient $\alpha_i$ (see also section 4).

The output from each sector $Q_j$, can be used either as a consumption good ($c_j$), or as an intermediate input in the production of the other sectors. Thus, the resource constraint of each sector $j$ is given by:

$$Q_j = c_j + \sum_{i=1}^{N} x_{ij}, \forall i = 1, ..., N.$$  

Consumption ($c_1, ..., c_N$) is combined to produce a single final good, according to the following function:\footnote{Alternatively, we could have used a utility function to generate the demand side of the economy, with our loss of generality.}:

$$Y(c_1, ..., c_N) = c_1^{\beta_1} c_2^{\beta_2} \cdots c_N^{\beta_N}.$$  

At this point, it is useful to note the Cobb-Douglas form of $Y(c_1, ..., c_N)$. This assumption will turn out to be important for the way labor resources are allocated across sectors in equilibrium (see section 3.1).

**Problem of the representative household**  This problem is quite trivial, but it is useful to write it down for future reference.

$$\max \{u(C)\} \quad \text{subject to} \quad C = wH + \Pi + T \quad (2)$$

where $C$ is aggregate consumption, $w$ is the price of labor, $\Pi$ are aggregate profits, and $T$ are transfers. These transfers are financed with the rents associated with the distortions that affect optimal decisions of firms (see below). Provided $u$ is increasing, the solution for this problem is trivial: the household will consume all the available income.

**Problem of the final good producer**  The problem of the final good producer consists on choosing $\{c_i\}$, taking $\{p_i\}$ as given, to solve:

$$\max_{\{c_i\}} \left\{ c_1^{\beta_1} c_2^{\beta_2} \cdots c_N^{\beta_N} - \sum_{i=1}^{N} p_i c_i \right\}.$$  

The first order conditions are given by:

$$\beta_i (Y/c_i) - p_i = 0 \iff \beta_i = \frac{p_i c_i}{Y}, \forall i.$$  

Just like in the textbook Cobb-Douglas utility maximization problem subject to a budget constraint, the first order conditions of the problem above imply that the consumption shares are constant and equal to the coefficient of each consumption good in the production (or utility) function.
Problem of the representative firm in sector $i$ There exists a representative firm in each sector. Each firm faces distortions that are specific to the industry. We assume three distortions: $\tau_i$, $\psi_i$, and $\phi_i$. The first distortion ($\tau_i$) represents output taxes that we will be able to pin-down using data on tax revenues at the industry level. The second distortion ($\psi_i$) enters in the firm’s problem in a way that resembles an output tax, but it is designed to capture other distortions that are not captured by the tax revenue data. In particular, this distortion introduces a wedge between marginal revenue and marginal cost. Under perfect competition marginal revenue and marginal cost are equalized, as a result, one of the forces behind this wedge is imperfect competition. However, other forces might act through the same channel, and be therefore captured by $\psi_i$. For simplicity we will refer to this wedge as the “markup” and will be defined in such a way that if $\psi_i > 1$, then it means that marginal revenue is above marginal cost, and vice-versa.

The last distortion, $\phi_i$, introduces a wedge between the value of the marginal productivity of labor and its marginal cost, and it enters in the firm’s problem as a labor tax. We define $\phi_i$ similarly to $\psi_i$, so that if $\phi_i > 1$, labor productivity is higher than the wage. For simplicity we will refer to this wedge as the “labor wedge”. Two alternative interpretations for this wedge are in place. The first one is that the marginal cost of labor faced by the firm is higher than the wage received by the workers due to policies and institutional constraints that make labor costs higher to firms. A second interpretation is that the value of the marginal productivity of labor is higher than the wage because of a low bargaining power of workers. The two interpretations differ in terms of who keeps the rents associated with the wedge. In the first interpretation, the rents are kept by agents involved in rent-seeking activities (not modeled), while in the second one are kept by the firms.

In the model it is assumed that the household is the owner of the labor resources, of the firms, and of any rents associated with wedges. As long as all rents are given back to the household as lump sum transfers, the results are independent of the above alternative interpretations.

The problem of the representative firm in industry $i$ is given by:

$$\max_{H_i, (x_{ij}), M_i} \left\{ \frac{(1 - \tau_i)}{\psi_i} - p_i A_i(H_i)^{\alpha_i(1-\sigma_i-\lambda_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i} - \phi_i w H_i - \sum_{j=1}^{N} p_j x_{ij} - p_{M,i} M_i \right\}$$

and the first order conditions (FOCs) are as follow:

$$\frac{(1 - \tau_i)}{\psi_i} \sigma_i p_i Q_i - p_i = \phi_i w, \forall i \quad (4)$$

$$\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i Q_i = p_{j,i}, \forall i, j \quad (5)$$

$$\frac{(1 - \tau_i)}{\psi_i} \lambda_i p_i Q_i = p_{M,i}, \forall i \quad (6)$$

The interpretation of the above conditions is straightforward, the household chooses labor, and intermediate inputs to equalize the (distorted) marginal revenue to the (distorted) marginal cost in each case. Note that the markup $\psi_i$ affects the three conditions above identically: it increases marginal revenue above marginal cost for each input; while the labor wedge $\phi_i$ affects only the first order condition associated with the choice of hours (4). This feature will be useful in the Calibration part in order to identify the value of these wedges.
Equilibrium With this, we can provide a definition of competitive equilibrium. Given import prices, taxes, and wedges \( \{p_{Mi}, \tau_i, \phi_i, and \psi_i\} \), a competitive equilibrium consists in quantities \( \{H_i, x_{ij}, M_i, c_i\} \); and prices \( \{p_j\} \) and \( w, \forall i,j = 1, \ldots, N \); such that:

1. \( \{c_i\} \) solves the representative final good producer problem at the equilibrium prices.
2. \( H_i, \{x_{ij}\} \) and \( M_i \) solve sector’s \( i \) producer problem at the equilibrium prices.
3. Markets for labor, and goods \( j = 1, \ldots, N \) clear.

A more operative definition of equilibrium is obtained by writing the production function as \( Q_i = A_i f(H_i, \{x_{ij}\}, M_i) \), where \( f(H_i, \{x_{ij}\}, M_i) = (H_i)^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i} \). Using this expression, an operative definition of equilibrium consists of quantities \( \{c_i, \{x_{ij}\}, H_i, M_i\} \), and prices \( \{p_i\}, w, \forall i, j \); such that:

\[
\frac{(1 - \tau_i)}{\psi_i} \alpha_i (1 - \sigma_i - \lambda_i) p_i A_i f(H_i, \{x_{ij}\}, M_i) = \phi_i w H_i, \forall i \tag{7}
\]

\[
\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i A_i f(H_i, \{x_{ij}\}, M_i) = p_j x_{ij}, \forall i, j \tag{8}
\]

\[
\frac{(1 - \tau_i)}{\psi_i} \lambda_i p_i A_i f(H_i, \{x_{ij}\}, M_i) = p_{Mi} M_i, \forall i \tag{9}
\]

\[
\beta_i = \frac{p_i c_i}{\sum_{i=1}^{N} p_i c_i}, \forall i \tag{10}
\]

\[
A_i f(H_i, \{x_{ij}\}, M_i) = c_j + \sum_{i=1}^{N} x_{ij}, \forall i \tag{11}
\]

\[
\sum_{i=1}^{N} H_i = H \tag{12}
\]

This constitutes a system of \( N^* N + 4N + 1 \) equations with the same number of unknowns, which has an analytic solution (see Jones, 2011, and the Appendix to this paper).

Note that the form of the resource constraint is related to the assumption on whether the rents from the distortions (\( \tau_i, \phi_i, and \psi_i \)) are given back to the household or not. For the baseline case, I assume that all rents from wedges and taxes are given back to the household, and therefore, \( T \) in the budget constraint 2 has three elements \( T = T^\tau + T^\phi + T^\psi \), which correspond to the aggregate rents associated with each distortion. As a result, these resources are available for consumption, and the resource constraints take the form in 11.

3.1 Analysis of equilibrium

In this section I would like to describe three features of the equilibrium that are important for the results in the paper.
Aggregate output and the vector of influence. The first feature is related to the way each sector is connected with the rest of the economy, and how this determines the effect that changes in productivity of a given sector has on aggregate outcomes. To start, note that it can be shown that equilibrium aggregate output is given by:

\[ Y = AH^\bar{\alpha} \]  (13)

where \( H \) is aggregate labor, \( \bar{\alpha} \) and \( A \) are constants that depend on parameters (see the Appendix). Furthermore, it can be shown that \( \ln A = m'a + \text{const} \), where:

\[ m'a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}, \]  (14)

\( a_i = \ln A_i, \forall i \), and the vector \( m \) is known as the “vector of influence” (Acemoglu et al., 2012) or the “vector of multipliers” (Jones, 2011b). The constant term \( \text{const} \) and \( \bar{\alpha} \) will differ between the distorted and the undistorted economies, but the vector of influence \( m \) will not. This vector is defined by \( m' = \frac{\beta'(I - B)^{-1}}{1 - \beta'(I - B)^{-1}\lambda} \), where \( \beta \) is the vector of consumption shares, \( B \) is the input-output matrix of technical coefficients with typical element \( \sigma_{ij} \), and \( \lambda \) is a vector with typical element \( \lambda_i \). The interpretation of an element \( m_i \) is that a 1% increase in productivity \( A_i \) rises aggregate GDP in \( m_i \%) \) (see Jones, 2011 for more details). In fact, this interpretation depends on the accuracy of the logarithmic approximation which it is only valid for small changes in \( A_i \). In general, and specially for the exercise of interest in this paper where closing productivity gaps requires large changes in \( A_i \), this interpretation will not be accurate.

To gain more intuition consider the case of a closed economy. In this case the vector of influence boils down to:

\[ m' = \beta'(I - B)^{-1} \]  (15)

Thus, the elements of this vector depend on two terms \( \beta \) and \( (I - B)^{-1} \). The first term measures the relative size of the sector in the economy as the share of sectoral consumption on aggregate GDP, while the second term is the Leontief inverse and gives a measure of interconnections, independent of size. Figure 7 presents a scatter plot of \( \beta_i \) and \( m_i \) for the Mexican sectors (see section 4, for details). Intuitively, the influence of a sector \( m_i \) is always bigger than \( \beta_i \) because the influence includes not only the effect of size, captured by \( \beta_i \), but also the effect of interconnections, captured by the Leontief inverse. Furthermore, in a closed economy, without distortions, the multipliers are equal to the Domar (1961) weights \( m_i = \frac{p_iQ_i}{Y} \), thus, the sum of the multipliers is bigger than one (see the appendix for a proof).

One useful expression follows from equation 13. Taking logs in both sides and deriving with respect to \( a_i \), we have:

\[ d\ln(Y) = m_i da_i \]  (16)

Which states that the log change in aggregate output is a linear function of the log change in productivity \( A_i \). The slope of this linear function is the multiplier of sector \( i \): \( m_i \). Since this multiplier varies across sectors, this implies that the linear function will differ across sectors too. I will go back to this relationship in Section 5, where I discuss the results.
Useful equilibrium relationships. A second feature relates to equilibrium relationships that are expressed in ratios instead of levels. This feature will be useful in the calibration and results sections. Notice that using equation 13, I can obtain expressions for the changes in each industry’s equilibrium gross output, and equilibrium aggregate GDP that result from changes in sectoral productivity $A_i$, and in distortions $\psi_i$, and $\phi_i$. Since the amount of labor in the whole economy is fixed the change in aggregate GDP will be equivalent to the change in GDP per worker. In particular, suppose that we change productivity of sector $i$ from $A^0_i$ to $A^1_i$, such that $A^1_i > A^0_i$, and we keep the productivity of all other sectors constant. Call $Q^1_i$ to the value of gross output of sector $i$ after the change in $A^1_i$ and call $Q^0_i$ to the value before the change. Similarly, let $Y^1$ be the value of aggregate GDP associated with $A^1_i$, and let $Y^0$ be the value for $A^0_i$. We show in the appendix that in equilibrium:

$$\ln \left( \frac{Q^1_i}{H^1_i} \right) = \ln \left( \frac{Q^0_i}{H^0_i} \right),$$

That is, the change in labor productivity of sector $i$ is proportional to the change in productivity $A_i$. For the counter-factual exercises performed in section 5, I take advantage of this relationship to avoid the computation of equilibrium levels. Thus, only changes in the equilibrium levels are computed.

Now consider the distorted economy in equations 1 through 6. In this case, we show in the appendix that equation 17 also holds for this economy, and in addition:

$$\ln \left( \frac{Y^1}{Y^0} \right) = f_\psi (\psi^0_i, \psi^1_i),$$

$$\ln \left( \frac{Y^1}{Y^0} \right) = f_\phi (\phi^0_i, \phi^1_i).$$

Which implies that we can compute the change in aggregate output associated with given changes in distortions. Notice that in contrast with the case of changes in the productivity parameter $A_i$, we do need to have both, the initial and the final levels for $\phi_i$ and $\psi_i$ in order to perform the above computations. Regarding the initial levels, in section 4, I describe the way in which these are calibrated. Then, in the counterfactual exercises of section 5, I will change the levels of these wedges to eliminate distortions in particular industries, and will make use of equations 18 and 19 to compute the effect of these changes in aggregate output.
Expenditure shares in equilibrium. The third important feature of the equilibrium is related to how the coefficients of the production function can be related to expenditure shares of firms. Consider the equilibrium allocation for an economy with no distortions, that is \( \tau_i = 0 \) and \( \psi_i = \phi_i = 1 \). Since the production function is Cobb-Douglas, we can relate expenditure shares to the coefficients. Equation 5 implies that \( \sigma_{ij} = \frac{p_j x_{ij}}{p_i Q_i} \), and, as a result \( \sigma_i = \sum_j \sigma_{ij} \) is the fraction of domestic intermediate inputs on gross output of industry \( i \):

\[
\sigma_i = \sum_{j=1}^{N} \sigma_{ij} = \sum_{j=1}^{N} \left( \frac{p_j x_{ij}}{p_i Q_i} \right) = \left( \frac{\sum_{j=1}^{N} p_j x_{ij}}{p_i Q_i} \right).
\]

Similarly, equations 5 and 6, imply that:

\[
\sigma_i + \lambda_i = \left( \frac{\sum_{j=1}^{N} p_j x_{ij}}{p_i Q_i} \right) + \frac{p_{M,i} M_i}{p_i Q_i}.
\]

In this undistorted economy, \( \sigma_i + \lambda_i \) is the share of intermediate inputs (domestic and imported) in gross output. This also implies that \( 1 - \sigma_i - \lambda_i \) is the share of value added in gross output. The reader is referred back to figures 4 and 5, where it was shown that there is a strong correlation between the share of value added in gross output of Mexico and the US, and that Mexico tends to have higher shares of value-added in gross output for the majority of the sectors with respect to the US. Taking the US as a relatively undistorted economy, it is possible to use equations 5, 6 and 21 to obtain estimates of the Mexican markups \( \psi_i \), \( \forall i \). More details of this strategy are provided in section 4.

3.1.1 The effect of distortions.

I divide the analysis on the effect of distortions in three parts. First, I analyze the effect of distortions on the allocation of labor across sectors; then, I move forward to analyze the effects of distortions on the allocation of output between final and intermediate uses; finally, I analyze the total effect of distortions on aggregate output. It will be convenient for didactic purposes to focus on the case of a closed economy facing wedges between marginal revenue and marginal cost (\( \psi_i \)).

Effect on labor allocations. In the undistorted economy, the equilibrium allocation of labor is determined by the equalization of marginal productivity (MP) of labor across sectors. What matters for this allocation is the way in which each unit of labor across the different sectors affects the supply of aggregate output \( Y \). To gain intuition, consider a simple 2-sector model without inter-sectoral linkages; thus, \( B = 0 \), \( \sigma_i = 0 \), \( \forall i \), and \( Q_i = c_i \), \( \forall i \). In this case, the trade-off is quite simple: the more labor is allocated to sector 1 and the more \( c_1 \) is produced; the less labor is allocated to sector 2, and the less \( c_2 \) is produced. The equilibrium allocation of labor is determined by the following efficiency condition:

\[
\left( \frac{\partial Y(c_1, c_2)}{\partial c_1} \right) \left( \frac{dc_1}{dh_1} \right) = \left( \frac{\partial Y(c_1, c_2)}{\partial c_2} \right) \left( \frac{dc_2}{dh_2} \right),
\]

where I have used a lower-case \( h \) to denote labor in this simple 2-sector model and make a difference with the labor allocation in the richer model with inter-connections. Note that I have arrived to the above equation by combining the first order conditions of the firm’s problem in each sector with the first order conditions in the problem of the composite producer. Alternatively, it can be derived as the optimal condition of a social planner’s problem. The left hand side is the marginal productivity of the composite with respect to labor \( h_1 \), while the right hand side is the marginal productivity with respect to \( h_2 \). The efficiency condition above indicates that labor should be allocated to sector 1 until
the marginal productivity of \( h_1 \) is equal to the marginal cost, which is precisely the lost production in sector 2 (due to the reduction in \( h_2 \)). In this simple model the efficient allocation of labor is given by:

\[
\frac{\hat{h}_i}{H} = \frac{\alpha_i \beta_i}{\sum_{s=1}^{N} \alpha_s \beta_s} = \frac{\alpha_i p_i Q_i}{\sum_{s=1}^{N} \alpha_s p_s Q_s},
\]

which depends on the influence of each sector (\( \beta_i \)) and the labor income shares (\( \alpha_i \)). Note that since \( B = 0 \), the vector of influence is simply \( m = \beta \). Also note that I have used a hat to indicate that this allocation corresponds to the undistorted economy. In addition, the second equality reveals that the fraction of labor allocated to sector \( i \) (\( \hat{h}_i/H \)) equals the share of labor compensations of sector \( i \) on aggregate labor compensations. Note that labor compensations in sector \( i \) are given by the fraction \( \alpha_i \) of the value added in sector \( i \), which in this economy is simply \( p_i Q_i \).

For the economy with inter-sectoral linkages a condition similar to 22 also holds, and it is easy to show that equilibrium labor is given by:

\[
\frac{\hat{H}_i}{H} = \frac{\alpha_i (1 - \sigma_i) m_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) m_s} = \frac{\alpha_i (1 - \sigma_i) p_i Q_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) p_s Q_s},
\]

The expression above says that labor ought to be allocated taking into account the relative influence of each sector (\( m_i \)), the share of labor income in value added (\( \alpha_i \)), and the share of value added in gross output (\( 1 - \sigma_i \)). In this network economy, the influence \( m_i \) takes into account the role of input-output relationships across sectors, as mentioned in the previous section. Note that, by the second inequality, relative labor also equals relative labor compensations, just as in the previous case. There is a slight difference, though, in the case of the economy with sectoral linkages, only a fraction \( (1 - \sigma_i) \) of gross output corresponds to value added, and labor compensations in sector \( i \) are given by \( \alpha_i (1 - \sigma_i) P_i Q_i \) (see equation 7). Finally, note that when there are no linkages \( (B = 0, \text{ and } \sigma_i = 0, \forall i) \), the expression above converges to equation 23.

The allocation of labor is independent of the productivity parameters \( \{A_i\}_{i=1}^{N} \). The reason for this is that there is some degree of complementary between any two consumption goods \( c_i \) and \( c_j \) in the production of the composite. To understand this, remember that the social planner wants to allocate labor in such a way that the marginal productivity of labor in the composite production function is equalized across sectors (equation 22). However, when \( A_i \) increases, this increases not only the marginal productivity of \( H_i \), but also the marginal productivity of \( H_j \) (because more \( c_i \) is produced and, thus, \( j \) has more \( c_i \) to produce with). It turns out that due to the Cobb-Douglas form of the production functions, the marginal productivity of both, \( i \) and \( j \), shift up by the same magnitude, and the allocation of labor remains unaltered in response to a change in productivity.

An important point regarding the literature on resource misallocation is opportune at this point. While in standard models of heterogeneous firms the allocation of resources is largely determined by relative productivity across firms; in standard multi-sector models, the allocation of resources across sectors is invariant to changes in productivity, and is largely determined by the vector of influence, which, in turn, is affected by the specification of demand (either through preferences or via the production function of the composite) and by the nature of the inter-sectoral network. As a result, misallocation across sectors will result different in nature than misallocation across plants.

\[8\]
From the perspective of the decentralized equilibrium, there are countervailing forces that affect the demand for labor in each sector in response to a change in productivity. For example, the increase in \( A_i \) increases demand for labor in sector \( i \) (a quantity effect), but the price of \( i \) decreases (due to increased supply) which tends to reduce the demand for labor (a price effect). Similarly, the demand for labor of the other sectors is also affected by opposite forces. At the end, wages and prices change in such a way that labor demands remain unaltered by the original change in productivity. Key in this mechanism is the fact that the cross-price elasticity of demand is zero when the production function of the composite is Cobb-Douglas.
Consider now the equilibrium allocation in the distorted economy, which is denoted without a hat to separate it from the allocation in the undistorted economy. It becomes relevant to distinguish between two cases regarding the distribution of rents from distortions:

**Case 1.- The rents are given back to the household as lump sum transfers.**

**Case 2.- The rents are lost in the sea.**

Take Case 1 and consider how the labor allocation looks like for the simple 2-sector model without inter-sectoral linkages and when only the markup wedge is present. In this case, the equalization of marginal productivity of labor across sectors is broken and the labor allocation is given by:

\[
\frac{h_i}{H} = \frac{\alpha_i \left( \frac{1}{\psi_i} \right) \beta_i}{\sum_{s=1}^{N} \alpha_s \left( \frac{1}{\psi_s} \right) \beta_s}, \quad i = 1, 2
\]

Thus, the presence of wedges in the simple economy creates misallocation of labor across sectors by shifting resources away from those sectors where wedges are large, and into those sectors where wedges are low. Note also that, there is no misallocation when wedges are the same across sectors \((\psi_i = \psi, \forall i)\). The reason for this is that in such case, marginal productivity is affected proportionally across sectors. Furthermore, in the simple model, the level of distortions does not affects aggregate output as long as wedges are homogeneous across sectors. This occurs in equilibrium because the only factor of production is inelastically supplied, and the wage rate absorbs all the burden imposed by distortions.

In the economy with inter-sectoral linkages, the distorted equilibrium allocation of labor is similar:

\[
\frac{H_i}{H} = \theta_i = \frac{\alpha_i (1 - \sigma_i) \left( \frac{1}{\psi_i} \right) m_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) \left( \frac{1}{\psi_s} \right) m_s} = \frac{\alpha_i (1 - \sigma_i) \left( \frac{1}{\psi_i} \right) p_i Q_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) \left( \frac{1}{\psi_s} \right) p_s Q_s},
\]

where \(\hat{m} = \beta'(I - \hat{B})^{-1}\), is a vector similar to the vector of influence but computed using the matrix \(\hat{B}\) instead, for which the typical element is \(\sigma_{ij}/\psi_i\). Note also that in this distorted equilibrium it is still true that the fraction of labor in sector \(i\) equals the share of labor compensations in that sector (second equality). The only difference now is that those compensations are affected by distortions. Thus, labor is misallocated away from those sectors with high markup wedges, and into sectors with low wedges.

Next, consider Case 2 which assumes that the rents from distortions are taken out of the economy. In this case, the markup wedge \(1/\psi_i\) is isomorphic to sectoral productivity \(A_i\). To see this, note first that, since \(T = 0\), the budget constraint in the household problem, 2, becomes \(C = \Pi + wH\), and markup rents are not available for consumption. This implies that the resource constraint has to be replaced with:

\[
\frac{1}{\psi_i} A_i f(H_i, \{x_{ij}\}, M_i) = c_j + \sum_{i=1}^{N} x_{ij}, \forall i
\]  

(24)

Note that if we replace equation 11 with equation 24, then both parameters \((1/\psi_i\) and \(A_i\) affect equilibrium conditions (7 to 10 plus 24) in exactly the same way. When the rents from distortions are not given back to the household, any changes in the value of \(1/\psi_i\), produce exactly the same equilibrium effects than changes in the value of \(A_i\). Given that -according to the above discussion-changes in productivity do not affect the allocation of labor in equilibrium, this means that distortions
in Case 2 do not misallocate labor across sectors neither. Nonetheless, changes in distortions can have large impacts on aggregate output through a feasibility channel, just as changes in productivity have it.

To further illustrate this point, take again the simple 2-sector model without inter-sectoral linkages, and make it even simpler by assuming that \( \alpha_i = 1 \), \( \forall i = 1, 2 \). In the absence of distortions, labor is given by \( h_i / \psi = \beta_i \), and aggregate output is \( \hat{Y} = (A_1 \beta_1) \hat{\psi} (A_2 \beta_2) \hat{\psi} H \). When distortions are introduced, and the rents from distortions are not given back to the household (Case 2), aggregate output is given by \( Y = (\hat{A}_1 \beta_1) \hat{\psi} (A_2 \beta_2) \hat{\psi} H \), where \( \hat{A}_i = \frac{A_i}{\hat{\psi}} \). Note that both, distortions and productivity, affect aggregate output in a similar way through \( A_i \).

The contrasting effects on aggregate output between Cases 1 and 2 is informative about the economic channel through which labor misallocation operates in the model. In particular, notice that the misallocation of labor is present in Case 1 due to the extra income effect that transfers entail. When we reduce the distortion of sector \( i \) (\( \psi_i \)) the rents associated with that distortion are also reduced, and, as a result, there is less income to consume, overall. This, in turn, translates into less labor being allocated to every sector. However, the reduction of \( \psi_i \) increases the marginal revenue product of labor in sector \( i \), which mitigates the negative income effect on that sector. These forces reallocate labor into sector \( i \), and away from every other sector. In the simple 2-sector model with distortions, when transfers are given back to the household (Case 1) the allocation of labor is given by \( h_i / \psi = \theta_i = \frac{\beta_i / \psi_i}{\sum_{s=1}^{N} \beta_s / \psi_s} \), and aggregate output is \( Y_1 = (A_1 \theta_1) \beta_1 (A_2 \theta_2) \beta_2 H \).

**Effect on the allocation of output between consumption and intermediates.** In the basic model without inter-sectoral linkages, distortions can’t have an impact on the supply of labor, because this factor is supplied inelastically. In the richer model with inter-sectoral linkages, there are \( N \)-inputs in addition to labor, which are provided using output from the sectors. In contrast to the supply of labor, the supply of these \( N \)-inputs is not inelastic and can be affected by the level of wedges. In fact, one important margin that is affected by the presence of the markup wedge is the allocation of gross output between consumption and intermediate uses.

To see this, note that the ratio of intermediate inputs to gross output is a function of distortions:

\[
\frac{X_i}{Q_i} = 1 - \frac{\beta_i}{\bar{m}_i (\psi_1, ..., \psi_N)},
\]

where \( \bar{m}_i \) is a typical element of \( \bar{m} = \beta' (I - \tilde{B})^{-1} \). Note that the elements \( \bar{m}_i \) depend negatively on the level of distortions (because a typical element of \( \tilde{B} \) is \( \frac{\sigma_i}{\bar{m}_i} \)) and that a single distortion \( \psi_i \) affects all elements of \( \bar{m} \), simultaneously. Thus, when we reduce a distortion, all \( \bar{m}_i \)'s increase and the fraction of gross output that is used as intermediate inputs increases in every sector. This result is intuitive, a reduction in \( \psi_i \) increases the ratio of expenditures in intermediate inputs over gross output in sector \( i \) because \( \left( \sum_{j=1}^{N} p_j x_{ij} \right) / p_i Q_i = \frac{\sigma_i / \psi_i}{\bar{m}_i} \) (see equation 5). As a result, intermediate demand increases for all sectors. Additionally, a reduction in \( \psi_i \), reduces the transfers associated with the rents, which affects the household’s demand for consumption. These effects combined lead to an increase in the ratios \( \frac{X_i}{Q_i} \), \( \forall j \).

**The total effect of eliminating the markup wedge.** The total effect of changing individual distortions on aggregate output can be first illustrated in the context of the 2-sector model. Denote values of the variables before the change in \( \psi_i \) with a 0 superscript, and those after the change with a 1 superscript. The effect of changes in \( \psi_i \) on \( \log \) aggregate output is given by:
\[ \ln \left( \frac{Y_i}{Y_0} \right) = \sum_{i=1}^{2} \beta_i \ln \left( \frac{\theta_{ij}}{\theta_{ii}} \right). \] (25)

As the above equation shows, a reduction in a single wedge \( \psi_i \) will not necessarily increase aggregate output. To see this mathematically, assume that we reduce the distortion of sector 1 \( (\psi_1 < \psi_1^0) \), then more labor will be allocated to sector 1 \( (\theta_{11}^1 > \theta_{11}^0) \), and less labor to sector 2 \( (\theta_{21}^1 < \theta_{21}^0) \). Since the total effect on aggregate output is equal to the sum of these two effects, it is not clear which one will dominate. In general, output will increase if the movement in \( \psi_i \) goes in the direction of equalizing wedges. Note that there is a slight difference between this term and the right hand side of equation 25: the weights of the relative \( \theta^i \)s are now given by the degree of influence \( (m_i) \), adjusted by the coefficient of labor in the production function \( \alpha_i(1 - \sigma_i) \). Finally, note the importance of the degree of influence to determine the final sign of the misallocation effect: if \( m_i \) is large, then the sum will give a larger weight to the positive effect \( (\theta_{11}^1 > \theta_{21}^0) \) and less weight to the negative one.

The second term in the equation above, is a direct effect of the change in markup \( \psi_1 \) on aggregate output. If the markup is reduced, this term is positive. This effect is present because the markup is affecting the supply of an input. We did not have this effect before, in the simple model, because the only input in the sectorial production functions was labor, and it was not being produced. Thus, the second term captures the common idea that less taxes/distortions on a factor, induce a higher supply of this factor.

The third term captures the effect of \( \psi_i \) on the degree of misallocation of gross output between final and intermediate uses. Note that the weights are the degrees of influence \( (m_i) \), adjusted by the shares of value added in gross output \( (1 - \sigma_i) \). As explained above, \( m_i \) controls the way in which gross output is divided between the two uses: consumption \( c_j \), vs., intermediate inputs \( X_j = \sum_{s=1}^{N} x_{sj} \). Thus, when \( \psi_i \) is reduced, \( m_i \) increases \( \forall j \), and the whole term is negative. The result is intuitive, since a reduction in the wedge translates into a lower ratio \( c_j/Q \), through this channel.

4 Calibration

The data used for calibration is available in the input-output tables of Mexico and the US published by the WIOD. The moments used for calibration are all expressed as shares to take advantage of the relationships between moments and parameters described in equations 3-6. These include: the share of value added in gross output, the share of domestic intermediate consumption in gross output, the share of imports of intermediate goods in gross output, and the share of purchases of intermediate
goods from sector \( j \) made by sector \( i \) in gross output of sector \( i \), which are also known as “technical coefficients” in the input-output tables.

The parameters of the model for both the US and Mexico are calibrated in two steps. First, I calibrate the model for the US assuming that it is a relatively undistorted economy. Second, using the US as a reference point, I calibrate the parameters and distortions for Mexico.

Starting with the parameters of the firm’s problem of industry \( i \), I assume that \( \tau_i = 0 \) and \( \psi_i = \phi_i = 1 \) in the US. There remain \( N + 3 \) parameters to calibrate (\( \{\sigma_{ij}\}_{j=1}^N, \sigma_i, \lambda_i, \alpha_i \)), so the \( N + 2 \) equations 4 to 6 along with the identity \( \sigma_i = \sum_{j=1}^N \sigma_{ij} \) are used to pin-down the value of these parameters.

Next, for the production function of the final good the value of \( \beta_i \) is needed. To calibrate this parameter I use equation 3 and data on consumption by industry. Consumption is defined as in the model: the difference between gross output of sector \( i \) and the value of purchases of sector’s \( i \) output made by the rest of the sectors.

With this at hand, only the set of exogenous relative prices of imports remains to be calibrated. To the best of my knowledge, there is no data on the relative prices of imported intermediate inputs across countries, which prevents the calibration of the full set of parameters and thus, it prevents the computation of equilibrium. This is the reason why, for the counterfactual exercises below, I rely on equations 17 to ?? to compute equilibrium changes and deliberately avoid the computation of equilibrium levels.

Measures of productivity gaps at the industry level are needed. In principle, I could use the production function in equation 1 to pin-down the value of \( A_i \). For this, I would need data on \( Q_i, x_{ij}, M_i \) and \( H_i \). Unfortunately, what is observed in the data is not \( Q_i \), but \( p_i Q_i \), and similarly for \( p_j x_{ij} \) and \( p_{M,i} M_i \). Thus, also need are the relative prices of gross output and imports at the industry level to perform this operation. This is a challenge since there are just a few sources available on relative prices across countries. In addition, prices are regularly collected on final goods and services, not on the output of industries as required by the model. One database available containing prices is the one used by Inklaar and Timmer (2013), who computed gross output prices for 35 industries departing from data on the prices of final goods and services. They implemented a methodology that includes the use of input-output tables to go from final good prices to industry output prices. To my knowledge, this is the only publicly available data set on gross output prices at the industry level that includes a comprehensive set of developed and developing countries. Using the prices from Inklaar and Timmer for Mexico and the US, and data on gross output and hours worked by sector from the WIOD, gross output labor productivity in sector \( i \) (\( Q_i/H_i \)) is computed. This estimation was already presented in Figure 2 of Section 2. Note that, in the model, total hours worked in the economy are exogenous, and also that the allocation of labor across sectors is invariant to changes in productivity \( A_i \) (see previous section). As a result, changes in equilibrium \( Q_i \) are equivalent to changes in equilibrium \( Q_i/H_i \), and thus equation 17 can be used to obtain the change in \( A_i \) that is necessary to close the observed labor productivity gap (\( Q_i/H_i \)).

Next, I proceed to calibrate the model to the Mexican economy. In order to measure distortions, two key assumptions are made regarding the value of the parameters of technology across the two countries. First, as is standard in the development literature, I assume that the parameter \( \alpha_i \) is the same in Mexico and the US, for all industries. This implies that, in the absence of distortions, the labor shares should be the same in the two countries, for all industries. Second, I assume that \( \sigma_i + \lambda_i \) is the same in both economies, for all industries. Similarly, this implies that in the absence of distortions, the share of value-added in gross output for a given industry, should be the same in both countries. As showed in Figure 4 of Section 2, there is a strong correlation in the shares of value-added in gross output between the two countries. Following the discussion in Sections 3 and 3.1, I will interpret the differences in value-added shares at the industry level between the two countries as the markup wedge. Furthermore, the reader should interpret the deviations from the 45 degree line in Figure 5 as the size of the distortions. In particular, I obtain the markup wedge, \( \psi_i \), in the following way:
Table 3: Calibrated parameters (averages)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.521</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.037</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.388</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.134</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.053</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.30</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.42</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.003</td>
</tr>
<tr>
<td>$m$</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Given the value of $\psi_i$, equation 4 and our assumption that $\alpha_i$ is common in Mexico and the US can be used to obtain the calibrated value of the labor wedge $\phi_i$. Notice that, the calibrated labor wedge depends on the value of the markup wedge. Put it differently, the observed labor income share in Mexico at a given industry, is affected by both: the markup wedge and the labor wedge. Thus, part of the explanation of the low labor income shares in Mexico relies on the existence of the markup wedges. This is the intuition used by Hall (1988), when he uses the ratio of labor compensation to total revenue to study imperfect competition in US industries.

Also, given the value of $\psi_i$, I can use equations 5, 6, and 3 to obtain the calibrated values for Mexico of $\{\sigma_{ij}, \sigma_i, \lambda_i, \beta_i\}$. It is emphasized that thanks to this calibration strategy, all parameters of the technology (for a given industry) are country-specific, except for two: $\alpha_i$ and $\sigma_i + \lambda_i$. Thus, the advantage of this strategy is that a significant amount of heterogeneity in the technologies of the two countries is still allowed. Table 3 shows the simple average across industries of the calibrated parameters.

5 Results

5.1 Key sectors: vector of influence and productivity gaps

The first set of results to report correspond to the calibrated vector of influence and the distortions for Mexico. Figure 8 reports the calibrated vector of influence. The multipliers of industries in services are typically large. Focusing on those industries with “influence” larger than 0.10, we see that 5 out of 7 are in services. The two non-services industries in this group are Construction and Food products, beverages and tobacco. Construction, is the one industry with the highest multiplier and it is typically considered manufacturing. However, in contrast to most manufacturing industries, construction is a non-tradable, a characteristic that shares with most service industries.

One simple approach to identify key sectors is to rank them in two dimensions: their relative productivity with respect to the US and its “influence”. Figure 9 presents this in a scatter plot. Notice that the measure of productivity used is the gross output labor productivity in Mexico relative to the US for each sector. The Figure additionally shows two straight lines drawn at the simple averages of the two variables. According to this ranking, the key sectors are the ones with labor productivity below average and multipliers above average, which correspond to those in the area located at the southeast of the intersection of the two straight lines. The sectors in this area are: trade, construction, transport, real estate activities, transport equipment, and agriculture. This simple approach gives a first impression on the relative importance of the sectors. However, the precise effect of productivity and influence has to be calculated using the model, because these interact in a non-linear way.
Figure 8: Calibrated vector of influence

Figure 9: Relative productivity and multipliers
5.2 Distortions

Figure 10 reports the value of $\psi_i$ for all the 33 sectors included in the analysis. Many values are close to 1, which imply no distortions. For the majority of the sectors, when the value of $\psi_i$ significantly differs from 1, it is in the direction that implies marginal revenue above marginal cost ($\psi_i > 1$). Nonetheless, some sectors show the opposite. To accurately interpret this last case, it is important to emphasize that, given the calibration strategy followed in the paper, distortions in Mexico are measured relative to the ones in the US. Thus, a markup less than 1 implies that the distortions faced in Mexico are smaller than the ones in the US. The unconditional average of industrial markups is 1.3, while if we only take the industries with markups above 1, the average is 1.6. Markups significantly above 1 are obtained in Water Transport, Wholesale Trade, Retail Trade, Food, Beverages and Tobacco, Other Non-Metalic Mineral, Wood Products, Business Services, Agriculture, Forestry & Fishing, Health and Social Work, Other Services, Inland Transport, Financial Intermediation, Hotels and Restaurants, Mining and Quarrying, Education, and Real Estate Activities.

Regarding the labor wedge, the estimates are presented in Figure 11. In this case, an overwhelming majority of the wedges are above 1. On average, the value of the marginal productivity of labor (net of the effect of markups) is 42% above the marginal cost of labor. Conditional on having a positive wedge, this number increases to 68%. Labor wedges are significantly above 1 for Other Services, Utilities, Transport Equipment, Retail Trade, Post and Telecom, Leather and Footwear, Wholesale Trade, Construction, Air Transport, Motor Vehicle and Fuel Trade, Basic and Fabricated Metal, Transport Services, and Electrical and Optical Eq.
5.3 Counterfactuals

5.3.1 Effect of closing gaps

With this at hand, it is now possible to perform counter-factual exercises. The first exercise I am interested in is closing productivity gaps of individual sectors to assess its effect on GDP and GDP per hour worked. As seen in section 3.1, the final effect of this not only depends on how lagged the sector is with respect to the US, but also it depends on how big the sector is in terms of its share in GDP, and how interconnected the sector is with the rest of the economy. Since there is high variation on the size of the gap, on sector’s weights, and on inter-connectedness, I expect to see large differences on the effects of each sector.

To perform this exercise I proceed in the following way. Given the value of relative productivity from the data, \( \ln \left( \frac{Q^{mx}_{i}/H^{mx}_{i}}{Q^{us}_{i}/H^{us}_{i}} \right) \), I use equation 17 to compute the change in \( A_i \) needed to close this gap. Then, I feed this estimated change in \( A_i \) into equation ?? to compute the associated change in aggregate GDP. The advantage of this procedure is that the computation of levels of the productivity parameter \( A_i \) is avoided. As explained above, finding the value of \( A_i \) is a difficult task given the presence of exogenous import prices. As expected, the larger is the gap in gross output labor productivity, the larger is the change in \( A_i \) needed to close this gap.

The effect in GDP associated with the elimination of each sectoral gap is presented in Figure 12. Closing the productivity gap in construction would increase aggregate output and aggregate labor productivity by around 20%! This is the sector that would give the biggest gain, but it is closely followed by: Wholesale Trade, Retail Trade, Food, Beverages and Tobacco, Agriculture, Forestry & Fishing, Business Services, and Real Estate Activities. Thus, once the model is used to assess the importance of each industry, the conclusion is that key sectors that give the biggest gains in aggregate
output, are mostly in services. Some non-services industries that also give large gains in aggregate output are: Food, Beverages and Tobacco, and Coke and Refined Petroleum, as well as Agriculture and Construction.

To illustrate the mechanics behind this result take two typical industries in manufacturing and services: Textile and Textile products (sector 4), and Wholesale Trade (sector 20), respectively. The industry of Textiles in the US is 8 times more productive than the corresponding one in Mexico, while Wholesale Trade is only 3 times more productive. However, Trade is not only a much bigger sector than Textiles in terms of its consumption share, it is also one of the most interconnected sector in the economy: the multiplier of Trade is 5 times bigger than the multiplier in Textiles. Therefore, closing the productivity gap in Trade gives much bigger gains in GDP per worker than closing it in Textiles (15% vs. 4% gains), despite the fact that the productivity gap is higher in Textiles.

One way to fully appreciate the importance of the variation in “influence” across sectors is by comparing the effect of closing productivity gaps when this feature is not present in the model. This exercise is presented in Figure 13 which plots the productivity gaps (in logs) in the x-axis, and the effect of closing the gaps in the y-axis (in logs). Remember from equation 16 that the change in log aggregate output is a linear function of the change in log individual productivity:

\[ d\ln(Y) = m_i da_i \]

The degree of influence is given by \( m_i \) which, in turn, depends on the consumption shares \( \beta_i \), and the interconnections captured by the Leontief matrix \( (I - B)^{-1} \) (see equation 15). Consider three different cases. In the first one, sectors do not differ in their consumption shares and there are no interconnections, that is: \( \beta_i = 1/N, \forall i \), and \( (I - B)^{-1} = I \). In this case \( m_i = 1/N \) and thus, the function above becomes identical for all sectors. In this case, as the markers in circles in Figure 13
show, the larger is the gap, the larger will be the associated change in log aggregate output when this gap is closed. In the second one, I allow for variation in the consumption shares and set the $\beta_i$'s equal to their calibrated values. Thus, the linear relationship between the size of the gap and the effect in $\log Y$, is now broken. This is represented with the plus (+) markers in the Figure. Finally, in the third case, I allow for both: differences in $\beta_i$ and inter-connections ($B \neq 0$). In this case, the correlation between log gap and the change in log $Y$ is even worse: the red stars (*) markers in the Figure represent this last case, these markers are all over the place, forming a cloud.

5.3.2 Effects of eliminating markup and labor wedges

The next counterfactual exercise of interest consists on reducing the distortions in the model: the markups and the labor wedges. It is convenient to split the analysis into the two previous cases: Case 1, when the rents from distortions are given back to the household; and Case 2, when the rents are lost. In both cases, I set distortions equal to 1, industry by industry, and compute its effects on aggregate output. This exercise is presented in Figure 14 for the case of the markup wedges.

In general, the effect of eliminating markups in Case 1 is smaller than the effect of eliminating productivity gaps, and, for the majority of the sectors, this effects is negligible. This is, in part, due to the fact that productivity gaps are larger than markup wedges, and also, to the fact that, as we explained in the previous sections, there exists countervailing effects on aggregate output in response to changes in markup wedges. If the markup of sector $i$ was above 1, then the effect of eliminating this markup is to increase the marginal revenue product in sector $i$, but to reduce the income available for consumption in all sectors. Thus, aggregate output could increase or decrease when a markup is eliminated.

A general rule is that output will increase if misallocation is reduced, which occurs when markup dispersion is lowered; and it will decrease otherwise. Also important for the effect on output is the degree of influence of the particular sector where the markup is being eliminated. The higher is the influence of the sector, the more important will be the direct effect of reducing the markup, and the less important will be the income effect associated with loosing the rents from the markup. For example, even when the markup in Wholesale Trade is smaller than the markup in "Other Services", eliminating the markup in the former gives bigger gains than eliminating it in the later. The reason for this is that the degree of influence of Trade is 5 times the degree of influence of Other Services. Consider one more example. Figure 10 shows that the two sectors with the largest markups are Education and

---

9There exists also a direct effect on the supply of the input, and an effect on the allocation of resources between final and intermediate uses (see section 3.1.1).
Real Estate. Note, however, from Figure 8, that Real Estate has one of the largest multipliers, while Education does not have a large one. Thus, the first term in equation 26 will be big for the case of Real Estate, while it won’t be as big (or even negative) for the case of Education. Consistent with these observations, Figure 14 shows that the effect of eliminating the markup wedge in education is negative, while the effect is positive for the case of Real Estate.

We also computed the effect in $Y$ of closing the labor wedge, under the assumption of Case 1. The results are presented in Figure 15. Similar to the the case of the markup wedge, there are also two opposing effects when eliminating the labor wedge. In general, the main message from Figure 15 is that the net effect of eliminating the labor wedge is small. We note, however, that there are important gains in reducing the labor wedge in trade, construction and the production of electrical and optical equipment.

Finally, I study the effect of eliminating the markup wedge in Case 2, when the rents from distortions are not given back to the household. This exercise is presented in Figure 16. As discussed in section 3.1.1, this assumption shuts down the extra income effect of Case 1, and makes the markup isomorphic to productivity $A_i$. As a consequence, the results of eliminating markups are much bigger than in Case 1. When all markup wedges are eliminated simultaneously, aggregate productivity increases 67.7%.

6 Conclusion

For a typical developing country, I have shown that once inter-sectoral linkages are taken into account, closing the productivity gap in an important number of services gives bigger gains in aggregate productivity than closing it in agriculture or in manufacturing, despite their larger gaps. This was done in the context of a general equilibrium framework with inter-sectoral linkages calibrated to Mexico.
Figure 15: Effect in $Y$ of reducing the labor wedges.

Figure 16: Effect in $Y$ of reducing markups under Case 2: when rents are not given back to the household.
and the US using input-output tables. Also, sector-specific distortions were computed: one similar to a markup which introduces a wedge between marginal revenue and marginal cost, and another one similar to a labor wedge that introduces a discrepancy between the marginal productivity of labor and the marginal cost of labor. I provided a quantitative assessment of the importance of these distortions for aggregate productivity.

The results suggest that analyzing distortions that lead to low productivity in services is a promising area of research in the development literature. The results also suggest that policies that tend to reduce the wedge between marginal revenue and marginal cost, in general, and for the labor market, can also increase measured productivity in a significant way. These policies include anti-trust reforms that aim to increase competition in product markets.

References


Appendix

Derivation of Equilibrium

In this appendix we follow closely Jones (2011b, b) and Acemoglu et al. (2012) in order to solve for equilibrium $Y$. We also show that changes in gross output and in aggregate output are proportional to changes in the exogenous productivity term $A_i$. In addition, we show that changes in aggregate output depend on the distortions in a non-linear way.

Consider the profit maximization for the composite:

$$\max_{\{c_i\}} \left\{ \prod_{i=1}^{N} c_i^{\beta_i} - \sum_{i=1}^{N} p_i c_i \right\}.$$

FOC:

$$\beta_i = \frac{p_i c_i}{Y}, \quad (27)$$
where $Y = \sum_{i=1}^{N} p_i c_i = \prod_{i=1}^{N} c_i^{\beta_i}$.

Next, consider the maximization problem for the representative firm in sector $i$:

$$\max_{H_i, (x_{ij}), M_i} \left\{ \frac{(1 - \tau_i)}{\psi_i} p_i A_i(H_i)^{\alpha_i(1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^\sigma M_i^\lambda - \phi_i w H_i - \sum_{j=1}^{N} p_j x_{ij} - p_{M,i} M_i \right\}$$

With first order conditions:

$$\frac{(1 - \tau_i)}{\psi_i} - \alpha_i(1 - \sigma_i - \lambda_i) p_i Q_i = \phi_i w, \forall i \quad (28)$$

$$\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i Q_i = p_j, \forall i \quad (29)$$

$$\frac{(1 - \tau_i)}{\psi_i} \lambda_i p_i Q_i = p_{M,i}, \forall i \quad (30)$$

Now, using the first order condition for $x_{ij}$ (equation 29) in the resource constraint for sector $j$, and multiplying both sides by $p_j$, we have:

$$p_j Q_j = p_j c_j + \sum_{i=1}^{N} \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i Q_i$$

Now, define $\gamma_i = \frac{p_i Q_i}{Y}$, and use equation 27 to obtain:

$$\gamma_j = \beta_j + \sum_{i=1}^{N} \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \gamma_i$$

and using matrix notation we have:

$$\gamma = \beta + \tilde{B} \gamma$$

$$\Rightarrow \gamma = \beta' (I - \tilde{B})^{-1}.$$  

where $\tilde{B}$ is an NxN matrix with typical element $\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij}$.

Next, using equations 28, 29, and 30, we write expression for $x_{ij}$, $M_i$ and $H_i$ in terms of $\gamma$. We will use these expressions later on, when we solve for $Q_i$ and $Y$.

$$x_{ij} = \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \frac{p_i Q_i}{p_j} = \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \frac{\gamma_i}{\gamma_j} Q_j$$  \hfill (31)

and
\[ M_i = \frac{(1 - \tau_i)}{\psi_i} \lambda_i \frac{p_i Q_i}{p_{M,i}} = \frac{(1 - \tau_i)}{\psi_i} \lambda_i \gamma_i \frac{Y}{p_{M,i}} \]  

(32)

\[ H_i = \frac{(1 - \tau_i)}{\psi_i} \alpha_i (1 - \sigma_i - \lambda_i) p_i Q_i \theta_i w = \frac{Y(1 - \tau_i) \alpha_i (1 - \sigma_i - \lambda_i) \gamma_i}{\psi_i \theta_i w} \]  

(33)

If we define \( H = \sum_{i=1}^{N} H_i \), we have:

\[ \frac{H_i}{H} = \frac{(1 - \tau_i) \alpha_i (1 - \sigma_i - \lambda_i) \gamma_i}{\sum_{j=1}^{N} (1 - \tau_j) \alpha_j (1 - \sigma_j - \lambda_j) \gamma_j} \equiv \tilde{\theta}_i \]  

(34)

and, \( \theta_i \equiv \frac{\psi_i}{(1 - \tau_i)} \). With this, we can use the above expressions for \( x_{ij} \), \( M_i \) and \( H_i \) into the production function \( Q_i \):

\[ Q_i = A_i H_i^\alpha (1 - \sigma_i - \lambda_i) \prod_{j=1}^{N} x_{ij}^\sigma M_i^{\lambda_i} \]

\[ \Rightarrow Q_i = A_i (\tilde{\theta}_i H)^\alpha (1 - \sigma_i - \lambda_i) \prod_{j=1}^{N} \left( \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \gamma_j (Q_j)^{\sigma_j} \right) \gamma_i \left( \frac{(1 - \tau_i)}{\psi_i} \lambda_i \gamma_i \frac{Y}{p_{M,i}} \right)^{\lambda_i} \]

\[ \Rightarrow Q_i = A_i \left( \frac{(1 - \tau_i)}{\psi_i} \right)^\alpha (1 - \sigma_i - \lambda_i) + \sum_{j=1}^{N} \sigma_{ij} \ln(q_j) + \lambda_i \ln(\frac{Y}{p_{M,i}}) + \sum_{j=1}^{N} \sigma_{ij} q_j \]

(35)

Taking logs of equation 35 above, gives us:

\[ q_i = \ln Q_i = \ln A_i + (\alpha_i (1 - \sigma_i - \lambda_i) + \sigma_i + \lambda_i) \ln \left( \frac{(1 - \tau_i)}{\psi_i} \right) + \alpha_i (1 - \sigma_i - \lambda_i) \ln \theta_i + \]

\[ \alpha_i (1 - \sigma_i - \lambda_i) \ln H + \sum_{j=1}^{N} \sigma_{ij} \ln(\frac{\gamma_j}{\gamma_i}) + \lambda_i \ln(\frac{Y}{p_{M,i}}) + \sum_{j=1}^{N} \sigma_{ij} q_j \]

Now, define \( a_i \equiv \ln A_i \), \( \delta_i \equiv \alpha_i (1 - \sigma_i - \lambda_i) \) and \( \text{const}_q_i \equiv (\delta_i + \sigma_i + \lambda_i) \ln \left( \frac{(1 - \tau_i)}{\psi_i} \right) + \delta_i \ln \theta_i + \)

\[ \sum_{j=1}^{N} \sigma_{ij} \ln(\frac{\gamma_j}{\gamma_i}) + \lambda_i \ln(\frac{Y}{p_{M,i}}) \] and write the above expression in vector notation:

\[ q = a + \text{const}_q + \delta \ln H + Bq + \lambda \ln Y \]  

(36)

This equation can be solved for \( q \) to yield:

\[ q = (I - B)^{-1} \{ a + \text{const}_q + \delta \ln H + \lambda \ln Y \} \]  

(37)

Finally, using the composite production function and the fact that \( \gamma_i = p_i Q_i / Y = \beta_i Q_i / c_i \), we have:

\[ \ln Y = \sum_{i=1}^{N} \beta_i \ln(c_i) = \sum_{i=1}^{N} \beta_i \ln \left( \frac{\beta_i Q_i}{\gamma_i} \right) = \sum_{i=1}^{N} \beta_i \ln \left( \frac{\beta_i}{\gamma_i} \right) + q = \sum_{i=1}^{N} \beta_i (\text{const}_c + q) \]
where \( const_c \equiv \ln\left(\frac{2}{m}\right) \). Now stacking this last equation into a vector we have:

\[
\ln Y = \beta'(const + q)
\]  

(38)

Using equations 37 and 38 we can find a solution for \( \ln Y \):

\[
\ln Y = \frac{\beta'const + \beta'(I - B)^{-1}\{a + const + \delta lnH\}}{1 - \beta'(I - B)^{-1}\lambda}
\]  

(39)

Which is precisely the desired equilibrium aggregate output in equation (8).

Next, I show that changes in gross output and in aggregate output are proportional to changes in the exogenous productivity. Consider a change in productivity of sector \( i \) from \( A_i^0 \) to \( A_i^1 \). Let \( Q_i^1 \) be the value of gross output of sector \( i \) after the change in \( A_i \) and \( Q_i^0 \) the value before the change, we will show that:

\[
\ln\left(\frac{Y_i^1}{Y_i^0}\right) \propto \ln\left(\frac{A_i^1}{A_i^0}\right)
\]  

(40)

and

\[
\ln\left(\frac{Q_i^1}{Q_i^0}\right) \propto \ln\left(\frac{A_i^1}{A_i^0}\right)
\]  

(41)

It is easy to show from equation 38 that

\[
\ln\left(\frac{Y_i^1}{Y_i^0}\right) = m_i (a_i^1 - a_i^0) = m_i \ln\left(\frac{A_i^1}{A_i^0}\right) \propto \ln\left(\frac{A_i^1}{A_i^0}\right)
\]

Taking the difference of equation 37 evaluated at \( A_i^1 \) and \( A_i^0 \), we have

\[
q_i^1 - q_i^0 = b_{ii}(a_i^1 - a_i^0) + \sum_{j=1}^{N} b_{ij}\lambda_j \ln\left(\frac{Y_i^1}{Y_i^0}\right)
\]

where \( b_{ij} \) is a typical element of \( (I - B)^{-1} \) and \( Y_i^i \) denotes that \( Y \) is evaluated at \( A_i^i \). Therefore,

\[
\ln\left(\frac{Q_i^1}{Q_i^0}\right) = q_i^1 - q_i^0 = (b_{ii} + \sum_{j=1}^{N} b_{ij}\lambda_j m_i)(a_i^1 - a_i^0) = (b_{ii} + \sum_{j=1}^{N} b_{ij}\lambda_j m_i)\ln\left(\frac{A_i^1}{A_i^0}\right)
\]

\[
\Rightarrow \ln\left(\frac{Q_i^1}{Q_i^0}\right) \propto \ln\left(\frac{A_i^1}{A_i^0}\right)
\]

The effect of changes in wedges on aggregate output and productivity can be derived similarly.
<table>
<thead>
<tr>
<th></th>
<th>Industry codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture, Hunting, Forestry and Fishing</td>
</tr>
<tr>
<td>2</td>
<td>Mining and Quarrying</td>
</tr>
<tr>
<td>3</td>
<td>Food, Beverages and Tobacco</td>
</tr>
<tr>
<td>4</td>
<td>Textiles and Textile Products</td>
</tr>
<tr>
<td>5</td>
<td>Leather, Leather and Footwear</td>
</tr>
<tr>
<td>6</td>
<td>Wood and Products of Wood and Cork</td>
</tr>
<tr>
<td>7</td>
<td>Pulp, Paper, Paper, Printing and Publishing</td>
</tr>
<tr>
<td>8</td>
<td>Coke, Refined Petroleum and Nuclear Fuel</td>
</tr>
<tr>
<td>9</td>
<td>Chemicals and Chemical Products</td>
</tr>
<tr>
<td>10</td>
<td>Rubber and Plastics</td>
</tr>
<tr>
<td>11</td>
<td>Other Non-Metallic Mineral</td>
</tr>
<tr>
<td>12</td>
<td>Basic Metals and Fabricated Metal</td>
</tr>
<tr>
<td>13</td>
<td>Machinery, Nec</td>
</tr>
<tr>
<td>14</td>
<td>Electrical and Optical Equipment</td>
</tr>
<tr>
<td>15</td>
<td>Transport Equipment</td>
</tr>
<tr>
<td>16</td>
<td>Manufacturing, Nec; Recycling</td>
</tr>
<tr>
<td>17</td>
<td>Electricity, Gas and Water Supply</td>
</tr>
<tr>
<td>18</td>
<td>Construction</td>
</tr>
<tr>
<td>19</td>
<td>Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel</td>
</tr>
<tr>
<td>20</td>
<td>Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles</td>
</tr>
<tr>
<td>21</td>
<td>Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods</td>
</tr>
<tr>
<td>22</td>
<td>Hotels and Restaurants</td>
</tr>
<tr>
<td>23</td>
<td>Inland Transport</td>
</tr>
<tr>
<td>24</td>
<td>Water Transport</td>
</tr>
<tr>
<td>25</td>
<td>Air Transport</td>
</tr>
<tr>
<td>26</td>
<td>Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies</td>
</tr>
<tr>
<td>27</td>
<td>Post and Telecommunications</td>
</tr>
<tr>
<td>28</td>
<td>Financial Intermediation</td>
</tr>
<tr>
<td>29</td>
<td>Real Estate Activities</td>
</tr>
<tr>
<td>30</td>
<td>Renting of M&amp;Eq and Other Business Activities</td>
</tr>
<tr>
<td>31</td>
<td>Education</td>
</tr>
<tr>
<td>32</td>
<td>Health and Social Work</td>
</tr>
<tr>
<td>33</td>
<td>Other Community, Social and Personal Services</td>
</tr>
</tbody>
</table>