From Firm Productivity Dynamics to Aggregate Efficiency

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Abstract

Lower financial development (i.e., lower access to credit) increases the dispersion of the marginal productivity of capital across firms (*misallocation*), therefore lowering aggregate production efficiency. However, models of firm dynamics with financial constraints generate modest losses due to misallocation relative to those found in the empirical literature, as pointed out by Midrigan and Xu (2013). For my version of this benchmark I compute losses of 3.8% of aggregate TFP for Colombia and 7.3% for Mexico. I revisit these results by constructing a quantitative model of firm dynamics with endogenous accumulation of firm productivity. In this model, financial constraints reduce the incentives of firms to invest in increasing firm productivity, reducing firm productivity growth. Additionally, for the firms that make investments to increase productivity, financial constraints become more persistent (compared to a model where firm productivity is purely stochastic). This channel amplifies the losses from misallocation to 15.8% for Colombia and 14.7% for Mexico. The model can partially account for the lower life-cycle productivity growth of firms in economies with underdeveloped financial markets and is consistent with more persistent constraints (measured through capital/output ratios) for the most productive units in a panel of manufacturing establishments for Colombia.

Keywords: aggregate productivity, financial constraints, firm productivity, technology adoption, misallocation.

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1 Introduction

Differences in total factor productivity (TFP) largely account for cross country differences in output per capita (Parente and Prescott, 2000; Caselli, 2005): controlling for different amounts of production inputs such as labor, human and physical capital, some countries are able to produce more goods and services than others. The empirical literature has documented that *misallocation* in developing economies can explain an important part of cross-country differences in TFP: in these economies there is more dispersion in the marginal productivity of inputs of production than in more advanced economies (all related literature is discussed below). This indicates that with the same total amount of inputs of production we could increase output by shifting resources from firms with low marginal productivity to those with higher marginal productivity.

The underdevelopment of financial markets has naturally been proposed as a source of misallocation: the lack of access to credit constrains firms from producing with the optimal level of capital. This implies that constrained firms with different levels of financial assets will have different marginal productivity of capital. However, recent work by Midrigan and Xu (2013) has pointed out that the misallocation losses generated by financial underdevelopment in a quantitative calibrated framework are modest. They find that in a country with no credit markets the losses are approximately 5% of TFP in their benchmark specification.[1]

I revisit this result by considering a model with financial constraints where firms are able to invest every period in order to increase their productivity (*knowledge capital*). I find that this channel amplifies the effect of financial constraints on misallocation. For Mexico, the model without endogenous firm-productivity accumulation generates misallocation losses of 7.3% in TFP, but this increases to 14.7% in a model with endogenous firm-productivity accumulation.

The amplification result is derived in part from a stronger covariance between firm productivity (which is endogenous in the model with knowledge capital) and output-capital ratios (higher for more constrained firms). In the model without knowledge capital, due to the fact that shocks are mean-reverting, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is constrained in one period can again be highly constrained in the following period if the endogenous productivity component increases.[2]

Financial constraints also have dynamic consequences by affecting firm productivity accumulation: constrained entrepreneurs invest less in knowledge capital

[1]To be more specific, they show that although financial constraints can have important quantitative effects on TFP, the impact is not generated through the misallocation channel. They also consider a one-time technology adoption decision without uncertainty, which can increase misallocation to 6.3% of TFP.

[2]In a different context, the role of the correlation of distortions with firm productivity was emphasized by Restuccia and Rogerson (2008) and Ranasinghe (2014). Hopenhayn (2012) discusses the specific conditions under which correlated distortions can generate a larger impact on TFP.

(firm productivity), establishing a link between firm productivity dynamics and aggregate production efficiency.[3] Previous work in development and financial frictions has mostly ignored the role of endogenous productivity growth of the firm or considered a one-time technological adoption choice (two exceptions are the recent work of Cole, Greenwood and Sanchez, 2012; and Caggese, 2014). The empirical literature has already stressed the relative lack of growth of firms in developing countries (see for example Hsieh and Klenow, 2012). The model with endogenous productivity accumulation can partially account for the lower life-cycle productivity growth of firms in an economy with underdeveloped financial markets.

How do financial constraints affect firm productivity growth? Investment in innovation is a costly and uncertain enterprize. As the capacity to obtain external funds is diminished, resources allocated to this effort will be reduced due to different mechanisms at work. First, the return of this investment in the case of success may be diminished by the inability to quickly increase production capacity if the credit necessary to do so is scarce (i.e., if entrepreneurs cannot rent the optimal level of physical capital). Second, financial constraints reduce profits obtained by entrepreneurs and therefore the amount of assets they are able to accumulate in every period. This can affect the amount of resources invested in new technologies.[4]

The empirical literature finds that innovation increases productivity and is therefore crucial for firm performance, whereas the lack of access to external finance constraints innovation and firm productivity growth and therefore reduces aggregate production efficiency.[5] Several studies have exploited cross-country firm-level data to analyze the role of financial constraints in determining innovation. Gorodnichenko and Schnitzer (2013) use the Business Environment and Enterprise Performance Surveys (BEEPS) of the World Bank, covering a wide array of sectors and countries with direct measures of innovation and financial constraints. They conclude that financial constraints restrain the ability of domestically owned firms to innovate and thus to reach the technological frontier. Furthermore, financial constraints are most detrimental for smaller and younger firms. They also document that financial restrictions at the firm level are strongly negatively correlated with aggregate measures of productivity as well as firm level TFP. In related work, Ayyagari, Demirgüç-Kunt and Maksimovic (2007), also find evidence that access to finance is an important determinant of innovation within a firm. Levine and Warusawitharana (2014) find a strong relationship between the use of external financing and future productivity growth in a large set of firms in four large European economies and find that the data does not support a reverse causality explanation.

[3]The theoretical and empirical literature on the relationship between financial development and economic growth underscores the role of better functioning financial systems in easing the external financing constraints that impede firm growth and innovation (Beck and Demirgüç-Kunt, 2006) and aggregate growth (Levine, 2005).

[4]A simple stylized model will illustrate these mechanisms.

[5]For survey-type treatments of the evidence of the positive impact of innovation on firm productivity and size-growth see Hall (2011) and Hall, Mairesse and Mohnen (2010). Although not without challenges, in this empirical literature innovation includes expenditures related to the introduction of new production processes, design and technical specifications, the implementation of new or significantly improved products (goods or services), new organizational methods in business practices, adoption and adaptation of existing technologies (Hall, 2011; Gorodnichenko and Schnitzer, 2013).

Additionally, financially underdeveloped economies will be characterized by a lower average ability of entrepreneurs, many of which have relatively low prospects of generating productivity growth through innovation. This is due to the lower demand for workers and the lower wages they receive, a larger mass of individuals opt to set up firms or become self-employed. These individuals, in the margin, tend to have lower ability to manage a firm and less incentives to invest in increasing firm productivity. This is exacerbated in developing countries by the existence of a large informal sector. This refers to entrepreneurs that do not register their firm in order to evade their tax obligations but have no access to credit (formal credit requires documentation). In developing economies a large part of the labor force belongs to small-scale, low-productivity and low-growth firms in this sector.[6]

The rest of the paper is organized as follows: Section 2 (S.2) overviews the related literature, S.3 provides the empirical motivation of this paper, S.4. discusses a stylized model of innovation and financial constraints, S.5 presents the quantitative framework, S.6 discusses the calibration of the model, S.7 presents the main results of the model, S.8 concludes.

2 Relation to the Literature

This paper builds upon several strands of the development and macroeconomics literature, a brief overview follows.

Misallocation. This literature refers to the large dispersion of marginal productivity of inputs observed across firms in a developing country, within narrowly defined industries. This finding suggests that great gains in aggregate output can potentially be generated by shifting production inputs from low marginal productivity firms to those with higher productivity. Hsieh and Klenow (2009), for example, find that gaps in the marginal products of labor and capital across plants can explain a large part of the differences in manufacturing TFP between China and India compared to the US. Busso, Fazio and Levy (2012) and Hsieh and Klenow (2012) perform similar empirical exercises for Mexico and Mexico-India, respectively. Busso, Madrigal and Pages (2012) compile evidence for 10 Latin American economies.

In terms of theoretical work, Restuccia and Rogerson (2008) analyze the potential quantitative effects of idiosyncratic tax schemes, suggesting the importance of evaluating specific distortions that affect the allocation of resources across firms. Financial frictions have been extensively studied in the development and firm dynamics literature; for recent quantitative examples see Amaral and Quintin (2010), Buera, Kaboski and Shin (2011), Arellano, Bai and Zhang (2012), Greenwood, Sanchez

[6]In the quantitative framework the informal sector is modeled to account for these facts, but all TFP and misallocation computations refer to formal sector firms. Employment in the informal sector is an enterprise-based concept and covers persons working in units that have informal characteristics in relation to, e.g., the legal status, registration, size, the registration of the employees, their bookkeeping practices, etc. (ILO).

and Wang (2013) and Steinberg (2013), among many others. My main contribution relative to this literature is to analyze the implications of considering the life-cycle productivity growth of firms and its interaction with financial constraints.

Bhattacharya, Guner and Ventura (2013) introduce non-stochastic accumulation of managerial skills (which determines firm productivity), while Gabler and Poschke (2013) allow firms to allocate resources to probabilistic *experimentation*, which in the case of success can lead to an increase in firm productivity. Similarly, Ranasinghe (2014) considers a setting where innovation carried out by firms determines the stochastic evolution of productivity. These authors evaluate the effects of distortions in the form of idiosyncratic taxes along the lines of Restuccia and Rogerson (2008). They show that assuming an exogenous distribution of firm productivity can lead to the underestimation of the consequences of distortions that affect the allocation of resources across production units.

Knowledge Capital. Different theories have linked the life-cycle growth of firms (or establishments) to the stochastic accumulation of knowledge specific to the production unit (Klette and Kortum, 2004; Atkeson and Kehoe, 2005). The theoretical framework analyzed here builds on that research. Doraszelski and Jaumandreu (2013) show that by introducing uncertainty in innovation they "(...) *allow shocks to accumulate over time, even firms with the same time path of R&D expenditures may not have the same productivity.*" They evaluate their model relative to the non-stochastic model of knowledge capital using a panel of manufacturing firms and find that a stochastic framework is favored by the data.

There is also research that emphasizes the role of intangible capital incorporated in the macro-neoclassical framework, such as Parente and Prescott (2000), McGrattan and Prescott (2005), Corrado, Hulten and Sichel (2009). This literature stresses the need to consider investment in intangibles such as software, R&D, as well as investments in building organizations to address many relevant questions in macroeconomics. Total business investment in intangibles has been found to be the dominant source of growth in labor productivity for the US (Corrado et al., 2009).

Informal Sector. There is a literature that analyzes the determinants of the size of the informal sector and its impact on aggregate outcomes as TFP and the size and productivity distribution of firms. Several results can be considered standard: the size of the informal sector decreases as the enforcement of financial contracts improves in the formal sector (Quintin, 2008; D'Erasmo and Moscoso-Boedo, 2012), increases with labor-market restrictions, heavier regulation of entry (for registered firms) and the tax burden of the formal sector and decreases with enforcement of legal obligations (Djankov et al., 2002; Perry et al., 2007; Leal Ordoñez, 2013). At the firm level, compliance with regulation is associated with better access to external finance, while informal sector firms are found to be less capital intensive, less productive, smaller and younger (Amaral and Quintin, 2006; Perry et al., 2007; Busso et al., 2012). For a more thorough discussion of the informal sector, see Lopez-Martin (2013).

3 Empirical Motivation

After a brief description of the data, this section documents the empirical evidence that motivates this study.[7] Relative to what has been documented for the US we can summarize the empirical facts for Colombia and Mexico as follows: (1) establishments grow less in terms of employment and productivity, (2) there is a larger share of employment in smaller establishments, (3) small establishments in the informal sector account for a large share of employment.

3.1 Data Description

For Colombia, the data is from the Annual Manufacturers Survey (AMS) for the period 1982-1998, constructed as a project of technical cooperation between the national statistics agency (DANE) and J. Haltiwanger (see Eslava et al., 2004). The AMS consists of an unbalanced panel of plants[8] with more than 10 employees or sales above a certain limit (approximately 35 thousand US dollars in 1998). The data-set includes information for each plant on output value and prices, input costs and prices, energy consumption in units and prices, number of production and non-production workers, book value of equipment and structures and four digits industry classification codes (CIIU). The AMS underwent changes in methodology of sampling and identification of plants, the creation of longitudinal linkages was necessary to consolidate plant identifiers through three different periods: 1982-1991, a transition period in 1991-1993 and 1991-1998. Plant-level TFP was generated through the estimation of a capital-labor-materials-energy production function (for details see Eslava et al., 2004).

For Mexico, the data is from the Economic Census 2009 conducted by the national statistics institute (INEGI). The census captures private establishments with a fixed location in urban areas and includes information on sales, workers, value added, value of fixed capital and labor remunerations, among other variables. It covered a total of 17.6 million workers in 3.6 million establishments in manufacturing, retail and wholesale and services (the figures for manufacturing are 4.6 and 0.4 million, respectively). In Mexico, total urban private employment reaches 33 million workers, the majority of those not captured by the Census belong to the informal sector and firms with less than 5 workers (Busso, Fazio and Levy, 2012). In spite of this limitation it is considered the most comprehensive dataset in Latin America (see Busso, Fazio and Levy, 2012; Hsieh and Klenow, 2012).

[7] Due to data availability and for better comparability, both with the literature and across the data-sets utilized here, we restrict our attention to manufacturing establishments. In Mexico, a very small share of firms has more than one establishment: 2.5% out of approximately 3.6 million firms (Busso et al., 2012). Additionally, it has been found that productivity at an establishment is positively related to the productivity of the firm to which it belongs (Bartelsman and Doms, 2000): well-run firms will be able to transfer technology, production methods, product designs and training across their production units. A large part of the literature uses the establishment as the unit of analysis. This approach is, at least in part, driven by data availability (Syverson, 2011).

[8] This data-set is also used in the cross-country study of firm dynamics by Bartelsman et al. (2009). Camacho and Conover (2010) analyze the dispersion of firm productivity applying the methodology developed by Hsieh and Klenow (2009).

3.2 Distribution of Employment and Establishments

Cross-country data shows that average size of both firms and establishments increase with income per capita and aggregate productivity[9] (Tybout, 2000; Alfaro, Charlton and Kanczuk, 2009; Garcia-Santana and Ramos, 2013).

Table 1. Distribution of Establishments and Employment by Size Class.

USA[1]	<5	5-19	20-99	100-499	≥500
establishments %	40.47	28.53	22.10	7.75	1.14
employment %	1.90	7.05	23.34	37.36	30.35
Mexico[2]	≤5	6-20	21-100	101-500	>500
establishments %	84.18	11.49	2.85	1.12	0.36
employment %	17.47	9.94	11.81	23.58	37.20

[1]Cole, Greenwood and Sanchez (2012), [2]Census INEGI (2009).

According to the Economic Census of Mexico of 2009, 27.4% of employment and 95.7% of establishments were accounted for by production units with less than or equal to 20 workers (Table 1). For the US, units with less than 20 workers account for 9% and 69% of employment and establishments respectively. These differences are maintained across different broad industry categories: manufacturing, retail and services (see the appendix for the size distribution by broad sectors for both establishments and employment).

These numbers are likely to understate the real differences due to the under-representation of small firms (Busso, Madrigal and Pages, 2012), particularly those in the informal sector.[10] Leal Ordoñez (2013), using micro-enterprize and census data, estimates that the informal sector accounts for 44-50% of employment.

In the case of Colombia, data is available for manufacturing establishments with over 10 workers. In 1998, the share of employment in firms with more than

[9]This is not without limitations. As is well known, the statistical under-representation of small firms (typically in the informal sector) in developing economies leads to understating the actual differences. An exception has been documented in European transition economies: plants with less than 20 employees account for a relatively small share of employment. This reflects the presence of large (formerly or still) state-owned firms inherited from the central planning period (Bartelsman et al., 2009).

[10]In the Appendix I provide a description of the main activities in the informal sector. It is worth noting that approximately 10% of the self-employed and micro-firm entrepreneurs were in that status due to loss of their previous job or because they could not find alternative employment (National Micro-Enterprise Survey of Mexico ENAMIN-2010), thus for the vast majority of the self-employed and micro-firm entrepreneurs this is a (self-reported) voluntary status.

500 workers is 25.6% (Camacho and Conover, 2010), while in the US, the equivalent figure (considering manufacturing firms with over 10 workers) is 31.7%. The informal sector in Colombia accounts for 52% of non-agricultural employment (ILO Statistics).

3.3 Establishment Life-Cycle Dynamics

This subsection documents the life-cycle growth of manufacturing firms in the US, Colombia and Mexico.[11] In the US most firms are born small: approximately 96.2 percent of firms that are 0-1 years have less than 20 workers.[12] Younger/smaller firms have higher exit rates, but those that survive tend to grow faster than older/larger firms (Klette and Kortum, 2004).

Table 2. The Life-Cycle of Establishments: Employment.

	USA[1]		**Colombia**[3]		**Mexico**[1]
relative size	surv.	all	surv.	all	all
age 5-9/age 1-4	1.6	2.0	1.5	1.4	1.4
age 10-14/age 1-4	2.0	3.0	1.7	1.7	1.5
age 15-19/age 1-4	2.3	4.1	1.8*	2.1*	1.6
	USA[2]		**Colombia**[3]		**Mexico**
growth in %	surv.		surv.	all	–
age 6/age 1	106.1		62.7	50.2	–
age 8/age 1	135.2		84.4	73.5	–
age 10/age 1	154.8		104.8	101.1	–

Source: [1]Hsieh and Klenow (2012), [2]Audretsch (1995), [3]computed w/AMS-DANE (1982-1998), *age 15-16 only.

Audretsch (1995) computes the average employment growth rates for 11,154 manufacturing firms established in 1976 for up to 10 years, we reproduce the results for the growth rates of surviving firms (data is from the Small Business Data Base of the US Small Business Administration). Hsieh and Klenow (2012) impute the life cycle from the employment growth from 1992-1997, comparing the average size of establishments within a given cohort grouped into five-year age bins (reproduced in

[11]Although international comparisons of firm data require caution and sometimes remain difficult to interpret, there is evidence of significant cross-country differences in firm-dynamics and post-entry performance. Differences in firm size are largely driven by within-sector differences and not by the sectoral composition of the economy (Bartelsman, Haltiwanger and Scarpetta, 2009). For example, four-digits industry effects account for less than ten percent of cross-section heterogeneity in output, employment and productivity growth rates across establishments (Foster et al., 2001).

[12]This is the average for the period 2000-2005. This group of firms (age 0-1 with less than 20 workers) accounted for an average 13.2 percent of total job creation in the same period compared to 8.4 percent for larger firms of age 0-1 (source: Business Dynamics Statistics, Census Bureau). New large firms are partly associated with new U.S. affiliates of foreign-owned firms.

Table 2).

Table 3. The Life-Cycle of Establishments: Productivity.

	USA[1]	**Colombia**[2]		**Mexico**	
relative avg.	all	surv.	all	all[3]	all[1]
age 5-9/age 1-4	1.6	1.0	1.0	1.4	1.5
age 10-14/age 1-4	2.1	1.2	1.1	1.5	1.6
age 15-19/age 1-4	2.8	1.3*	1.2*	1.5	1.6

Source: [1]Hsieh and Klenow (2012),
[2]computed w/AMS-DANE (1982-1998), *age 15-16 only,
[3]computed w/INEGI Census (2009), cross section.

The life-cycle growth of establishments for Colombia is computed using the AMS panel database for the period 1982-1998. We are able to follow establishments up to age 16 (we cannot impute the age of establishments born in 1982 or earlier). For the growth rate of *all* establishments in the lower panel first I compute, in every year, the average size of all establishments of a particular age. Then I calculate the growth rate of this average for each cohort. Finally, for each age I take the median across cohorts. For example, to calculate growth at age 4, I have 13 observations representing cohorts of establishments born between 1983 and 1995. There is variation across cohorts, taking the average across cohorts instead of the median results in slightly lower life-cycle growth. To compute the growth of *survivors* in the lower panel, I compute the growth of each individual establishment at each age, I take the average of establishment growth within a cohort and then the median across cohorts (Table 2). This procedure is equivalent to the one in Audretsch (1995) but repeated for different cohorts.

For the upper panel of Table 2, first I take the average of all establishments age 1-4 for each given year. To calculate the relative size at age 5-9, I can start in 1991 (the first year where we have establishments of age 9) resulting in 8 observations, for age 10-14 we can start in 1996, resulting in 3 observations. This procedure is comparable to that in Hsieh and Klenow (2012) but repeated for different cohorts. For Colombia, Table 3 uses firm TFP computed by Eslava et al. (2004).

For Mexico, we have data available from the 2009 Economic Census (a cross section). Hsieh and Klenow (2012) are able to use the Census data for 1999, 2004, 2009, which allows them to track cohorts for up to 10 years. It is not possible to compute statistics for *survivors* since there is no information to link establishments across time. We replicate their results for the life-cycle growth in terms of employment. For firm level TFP, in the case of Mexico, we compare their results with estimates from the cross section (Table 3).

4 Stylized Model of Innovation and Financial Constraints

In this section a stylized two-period model is presented to highlight the interaction between financial constraints and innovation along the intensive and extensive margins.[13] The intensive margin considers how financial constraints affect innovation for a firm with a given productivity level. The extensive margin refers to the impact on firms with different productivity: general equilibrium effects may lead to changes in the composition of firms.

For simplicity I assume that in the first period no production takes place, the entrepreneur is endowed with financial assets $b > 0$ which can be allocated to consumption c in the first period, to savings b' for the second period (in this section we assume there is no interest rate on savings), or invested in the innovation good x. In this set-up, innovation investment is fully financed with internal funds (evidence supporting this assumption is discussed below).

In the second period knowledge capital can take low and high levels, $n \in \{\underline{n}, \overline{n}\}$ respectively, determining the production possibilities. There is a stochastic innovation technology that determines the probability $P(\overline{n} \,|\, x) \in [0, 1]$ depending on the amount invested in the innovation good x. This function is increasing and concave in x.

Production takes place in the second period. At this point the entrepreneur needs to rent capital k at a cost equal to the interest rate r plus the physical depreciation rate of capital δ. The rental of capital is subject to an exogenous collateral constraint $k \leq \psi\, b'$, where $\psi \geq 1$ is a parameter that determines the ability to collateralize financial assets. After production takes place, consumption for the entrepreneur results from profits of the firm and savings. Consumption is valued in both periods through a standard utility function $u(c)$ and discounted in the second period by β. The production technology is given by $n^{1-\nu} k^{\nu}$ with $\nu \in (0, 1)$.

In the second period, the profits of the firm given knowledge capital n and assets b' are:

$$\pi(n, b') = \max_{\{k\}} n^{1-\nu} k^{\nu} - (r + \delta)\, k \quad \text{s.t.} \quad k \leq \psi\, b'$$

The intertemporal problem of the entrepreneur is to select consumption, savings and investment in innovation to maximize expected discounted utility:

$$\max_{\{x, c, b' \geq 0\}} u(c) + \beta \sum_{\{n'\}} P(n' \,|\, x)\, u\left(\pi(n', b') + b'\right) \quad \text{s.t.} \quad c + x + b' = b$$

[13]Alternative prototypical models of this interaction, with a complementary focus on liquidity shocks and the cost of external finance, are discussed in Gorodnichenko and Schnitzer (2013).

The intertemporal optimality equation for assets b' is given by:

$$u_c(c) = \beta \sum_{\{n'\}} P(n' \,|\, x)\, u_{c'} \left(\pi(n', b') + b'\right) \left(\pi_{b'}(n', b') + 1\right)$$

Where u_c refers to marginal utility and $\pi_{b'}$ is the derivative of profits in the second period with respect to assets. This derivative will be positive when the collateral constraint is binding. In addition to the standard consumption smoothing motive for savings, there is an incentive to save to relax the collateral constraint in the second period.

Consider the function $P(\overline{n} \,|\, x) = \zeta\, x^{\lambda}$ with parameters $\lambda \in (0, 1)$ and $\zeta > 0$, the intertemporal optimality equation for innovation investment x is (in an interior solution):

$$u_c(c) = \beta\, \zeta\, \lambda\, x^{\lambda-1} \left(u(\pi(\overline{n}, b') + b') - u(\pi(\underline{n}, b') + b')\right)$$

We are interested in understanding how financial constraints affect investment in innovation. The left hand side on the intertemporal optimality condition of innovation investment does not depend directly on ψ. The derivative of the right hand side, defining $\Delta u = u(\pi(\overline{n}, b') + b') - u(\pi(\underline{n}, b') + b')$ is:

$$\frac{\partial \Delta u}{\partial \psi} = u_{c'} \left(\pi(\overline{n}, b') + b'\right) \frac{\partial \pi(\overline{n}, b')}{\partial \psi} - u_{c'} \left(\pi(\underline{n}, b') + b'\right) \frac{\partial \pi(\underline{n}, b')}{\partial \psi}$$

where it is always the case that:

$$\frac{\partial \pi(\overline{n}, b')}{\partial \psi} \geq \frac{\partial \pi(\underline{n}, b')}{\partial \psi}$$

with strict inequality when the collateral constraint is binding (it can be binding either in both states or in the high knowledge capital state). If the constraint is only binding in the high knowledge capital state, then $\partial \Delta u / \partial \psi > 0$: this implies that relaxing the collateral constraint promotes innovation investment.

With logarithmic utility it can be proven that $\partial \Delta u / \partial \psi > 0$, which again implies that relaxing the collateral constraint promotes innovation investment. The same result holds with $u(c) = c$, risk neutral preferences.[14] With preferences $u(c) = c^{1-\sigma}/(1-\sigma)$, under some parameterizations (in particular relatively high σ), it is possible for innovation investment to be decreasing in ψ. Innovation investment also depends positively on initial assets. Note that in a dynamic model financial wealth is an endogenous state variable and the entrepreneur may outgrow the collateral constraints by saving.

[14] Caggese (2012) finds that uncertainty (as measured by the volatility in the sectoral profits-assets ratio in a panel of manufacturing firms) reduces the innovation investment of entrepreneurial firms (financially more undiversified) but not the innovation of non-entrepreneurial firms. These results correspond to innovation related to the production of new products, which is linked to increased uncertainty for the firm. The negative impact of uncertainty on innovation is larger for less diversified firms.

Suppose now that there are individuals with heterogeneous entrepreneurial ability, which affects the production technology of the firm they manage. The production function is $(\varphi n)^{1-\nu}k^{\nu}$ where φ, the entrepreneurial ability, varies across individuals. To isolate the role of the extensive margin consider a risk neutral utility function and no collateral constraint. The static profit maximization problem is given by:

$$\pi(\varphi\, n) = \max_{\{k\}} \, (\varphi n)^{1-\nu}\, k^{\nu} - (r+\delta)\, k$$

The inter-temporal problem, simplified to isolate the role of the extensive margin, is now given by:

$$\max_{\{x\}} \, -x + \beta \sum_{\{n'\}} P(n' \,|\, x)\, \pi(\varphi\, n')$$

With a small amount of algebra it can be shown that the optimal first order condition for innovation investment in an interior solution is:

$$x^{1-\lambda} = \beta\, \zeta\, \lambda\, \varphi\, (\overline{n} - \underline{n})\, (1-\nu)\, (\nu/(r+\delta))^{\nu/1-\nu}$$

This condition implies that x is increasing in φ when $\nu < 1$. In the quantitative model financial constraints lower the demand for labor resulting in lower wages. This leads to individuals with lower entrepreneurial ability φ to set-up a firm.

5 Quantitative Model

The model builds upon the frameworks of occupational choice and heterogeneous entrepreneurial ability[15] of Lucas (1978) and industry dynamics of Hopenhayn (1992). There is a continuum of individuals who possess heterogeneous innate entrepreneurial ability and every period decide whether to be workers or establish a firm and become entrepreneurs. The operations of the firm are subject to transitory stochastic shocks which are observed at the beginning of each period, before production and occupation decisions are made. All individuals earn the same wage as workers, since there is no heterogeneity in their effective units of labor and workers are perfectly mobile.[16]

The firm is a *storehouse of information* (Prescott and Visscher, 1980; Atkeson and Kehoe, 2005), or *knowledge capital* (Klette and Kortum, 2004; Corrado et al. 2009). Entrepreneurs in the formal sector can, while the firm is in operation, allocate resources to investment in technology through a controlled stochastic process. Innovation is an uncertain enterprize, as in Klette and Kortum (2004) and Atkeson and Burstein (2010): entrepreneurs decide every period the amount of resources devoted to improving firm productivity, which determines the probability of an increase in

[15] Differences in management quality are an important determinant of productivity differences across firms (see Bartelsman and Doms, 2000; Foster et al., 2001; Syverson, 2011).

[16] The evidence on whether labor markets are segmented across informal and formal sector firms suggests mixed results at best, see the discussion in Perry et al. (2007, Ch. 3 and 4).

firm productivity. Knowledge capital summarizes the history of past investment and innovation success of the firm.[17]

The entrepreneur, who is both owner and manager of the firm, can opt to conduct operations in the formal or informal sector. The trade-off is the following: formal sector firms have to pay an initial registration cost and taxes but they have better access to external finance. Informal sector firms do not pay taxes or the initial registration cost, but have no access to external finance and cannot accumulate knowledge capital. Additionally informal sector firms face a specific convex cost of production. This cost represents the inability to engage in legal contracts, the cost of enforcing property rights when not protected by the government and worse access to infrastructure facilities and services, etc. (Perry et al., 2007). Entrepreneurs may first establish their firm in the informal sector and later transition to the formal sector but a formal sector entrepreneur may not switch directly into the informal sector.

We can start introducing notation by letting $s = (\varphi, n, a, b)$, where φ is the individual's permanent entrepreneurial ability, a is a transitory productivity shock, b are financial assets and n is knowledge capital. Additionally $z \in \{i, f, w\}$ denotes whether the individual is an entrepreneur in the informal or formal sectors or a worker, respectively.

5.1 Preferences

Time is discrete and a period, indexed by t, represents a year. Individuals value the consumption of the final good, denoted c_t, through lifetime and intratemporal preferences represented as follows:

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} (\beta\,(1-\mu))^t\, u(c_t)\right] \quad \text{and} \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{1}$$

where β is the discount factor, σ is the coefficient that governs risk aversion. The probability that an individual dies in every period is μ, so that the *effective* discount factor is $\beta\,(1-\mu)$. When an individual dies, his assets disappear and he is immediately replaced by another individual with the same entrepreneurial ability φ so that the mass of individuals and their distribution over ability is constant (the rest of the initial state variables are specified below).

5.2 Production Technology

In this economy production of the final good is carried out by single establishment firms and each firm is managed by its owner/founder. Individuals posses innate and permanent entrepreneurial ability φ received according to a distribution $h(\varphi)$.

[17] Klette and Kortum (2004) extend the endogenous growth literature by introducing research in incumbent firms. Aghion, Howitt and Mayer-Foulkes (2005) is a relatively recent example of the endogenous growth literature, where the impact of financial frictions on economic growth is assessed.

Their operations are subject to productivity shocks a that follow an AR(1) process discretized in a Markov matrix denoted $\Lambda(a' \,|\, a)$. Additionally, entrepreneurs are able to accumulate knowledge capital denominated n (through a process described below) and have access to a decreasing returns to scale production technology (in terms of capital and labor) that is common across sectors:

$$q = e^{a} \, (\varphi \, n)^{1-\nu} \, f(k,l)^{\nu} \quad \text{with} \quad f(k,l) = k^{\alpha} \, l^{1-\alpha} \tag{2}$$

where k is capital and l is labor used in production. Following Lucas (1978), we call $\nu < 1$ the span-of-control parameter that determines the decreasing returns to scale (with respect to capital and labor) in the production technology.

5.3 Innovation Technology

Entrepreneurs can invest in the innovation good x to increase the stock of knowledge capital.[18] This good has price p_x.[19] Three outcomes are possible every period, depending on the amount of investment in the innovation good in the previous period: knowledge capital may increase by a proportion Δ, it may remain constant, or decrease by Δ. Knowledge capital is defined on the grid $\{\underline{n}, \underline{n}\,(1+\Delta), \underline{n}\,(1+\Delta)^2, \ldots, \overline{n}\}$, where $\underline{n}$ and $\overline{n}$ are the lowest and highest possible levels of knowledge capital, respectively.[20]

The probability of a successful outcome is given by:

$$P(n' = n\,(1+\Delta) \,|\, n, x) = \zeta \; (x/n)^{\lambda} \; (1-\varepsilon) \tag{3a}$$

subject to the following conditions:

$$\zeta \; (x/n)^{\lambda} \in [0,1] \quad \text{and} \quad \{\lambda, \varepsilon\} \in [0,1) \tag{3b}$$

There are diminishing returns to innovation investment x. Fixing a probability of success in innovation, $P(n\,(1+\Delta) \,|\, s, x)$, the necessary investment in innovation goods x to increase the size of the firm by a fixed percentage is proportional[21] to knowledge capital n. The probability of the worst outcome:

$$P(n' = n/(1+\Delta) \,|\, n, x) = \varepsilon \tag{3c}$$

This shock represents negative events not influenced by firm decisions (obsolescence of products, loss of markets to the competition, etc.). Knowledge capital

[18] The stochastic innovation process specified in this section builds on those considered by Klette and Kortum (2004), Atkeson and Burstein (2010).

[19] We will initially consider the case where the innovation good is produced with the final good and set $p_x = 1$. This can be extended to consider that innovation requires labor.

[20] The model can be extended to consider unbounded knowledge capital, which would require additional conditions to guarantee a well defined dynamic program and convergence in the stationary distribution (see Atkeson and Burstein, 2010).

[21] It can easily be verified that optimal labor and capital inputs, output and profit are proportional to knowledge capital n under the production function previously specified in the case of no financial restrictions and the unconditional mean value of the stochastic shock, $e^{\overline{a}} = 1$.

summarizes the history of investment and success in innovations and governs the size of the firm (as in Klette and Kortum, 2004). Furthermore, it is lost when the firm closes (regardless of whether exit is due to an exogenous exit shock or the entrepreneur finds it optimal to close the firm). Knowledge capital is assumed to be completely firm-specific and there is no market for its trade.

5.4 Workers

The problem of the worker amounts to a savings decision and determining the conditions under which it is optimal to establish a firm:

$$\begin{aligned} v_w(s) &= \max_{\{b' \geq 0\}} u(c) + \beta\,(1-\mu) \sum_{\{a'\}} \Lambda(a' \,|\, a)\, v(s') \\ \text{s.t.} \quad c + b' &= w + (1+r)\, b \quad \text{and} \quad n = \underline{n} \end{aligned} \tag{4}$$

At the beginning of each period, after observing the transitory productivity shock a, workers face their occupational choice:

$$v(s) = \max\{v_i(s), v_f(\varphi, \underline{n}, a, b - c_e), v_w(s)\} \tag{5}$$

The worker is free to continue in the labor market and earn wage w every period, become an informal sector entrepreneur (represented by the value v_i), or a formal sector entrepreneur, which requires paying the fixed registration cost c_e (this value is represented by v_f).

Whenever individuals re-enter the labor market, their knowledge capital is reset to $\underline{n}$, this underscores the interpretation that it represents an intangible asset embedded in the firm. Workers are not able to invest in innovation. We abstract from labor-income risk.[22] All new-born individuals receive an entrepreneurial ability φ from the distribution $h(\varphi)$, transitory shock a from its unconditional distribution and initial assets $\underline{b}$ (set equal to zero in the baseline model).

Occupational choice depends on the ability of the individual as an entrepreneur but also on financial wealth, necessary to register the firm or to reach a profitable scale when financial constraints are present. Figure 2 depicts the optimal occupation choice (5) of a worker for a fixed level of productivity shock a, in an economy with a large informal sector such as Mexico or Colombia. The graph has entrepreneurial ability φ on the x-axis and b/w (financial assets normalized by the wage) on the y-axis.

5.5 Formal Sector Entrepreneurs

Given the choice of labor and capital input, profits for a formal sector en-

[22] Labor income risk is an important factor in models of interest rate determination, we will abstract from a complete model of the interest rate and set it exogenously (a standard small-open economy assumption).

trepreneur are given by:

$$\pi(s,f) \;=\; q - (\delta + r)\, k - w\, l$$

To register in the formal sector, entrepreneurs have to pay a fixed cost c_e. Once in the formal sector, the entrepreneur may go back to being a worker and c_e is lost. Additionally, the entrepreneur cannot transition directly to the informal sector:

$$v_f(s) = \max_{\{l,k,x,b' \geq 0\}} u(c) + \beta\,(1-\mu) \sum_{\{a',n'\}} \Lambda(a' \,|\, a)\, P(n' \,|\, n, x)\, \max\{v_w(s'), v_f(s')\}$$
$$\text{s.t.} \quad c + b' = \;(1-\tau)\;(\pi(s,f) - p_x\, x) + (1+r)\, b \quad \text{and} \quad k \;\leq\; \overline{k}(s,f) \qquad (6)$$

where τ are taxes to profits. The entrepreneur is able to invest in the knowledge capital of the firm as long as the formal firm is active, but is lost if the individual decides to return to the labor market. The choice of capital input is restricted by an endogenous collateral constraint, to which we turn shortly. The firm dies with the entrepreneur.

5.6 Financial Markets

In specifying the endogenous collateral constraints I follow Amaral and Quintin (2010) and Buera et al. (2011). At the beginning of a period the entrepreneur makes a deposit b and rents capital k from a financial intermediary. At the end of the period, the entrepreneur receives his deposit earning interest rate r and pays the cost of capital rental at the total rate of $r + \delta$. Borrowing and capital rental are realized within a given period and the assets of the individual are restricted to be positive $b \geq 0$ in all periods.

Entrepreneurs may renege on financial contracts after production has taken place. If this occurs, the entrepreneur keeps a fraction $(1-\psi)$ of the un-depreciated capital and the revenue net of labor and tax payments.[23] The punishment for default is the loss of the financial assets deposited with the financial intermediary b. Entrepreneurs regain access to financial markets in the following period without additional costs. This implies that a static condition determines enforceable allocations, allowing for the consideration of financial constraints in a tractable manner. In this setup parameter ψ indexes enforcement of financial contracts in the economy, which encompasses economies with no credit $\psi = 0$ and perfect credit markets $\psi = 1$.

The analysis is restricted to financial contracts that are incentive-compatible, there is no default in equilibrium. Effectively, imperfect enforcement of financial contracts determines an upper bound $\overline{k}(s,f)$ on the amount of capital that entrepreneurs are able to borrow.

Mathematically the financial constraint can be described as follows. In the case of no-default the entrepreneur receives profits net of taxes, plus interest rate

[23] I assume that the entrepreneur cannot avoid paying taxes in the event of default.

income from financial assets:

$$\max_{\{l\}} (1-\tau)\,(q - w\,l - (r+\delta)\,k - p_x\,x) + (1+r)\,b \tag{7}$$

In the case of default the entrepreneur would receive (off-equilibrium):

$$\max_{\{l\}} (1-\psi)\,\left((1-\tau)\,(q - w\,l) + (1-\delta)\,k\right) - (1-\tau)\,p_x\,x \tag{8}$$

Capital rental is said to be enforceable if and only if it satisfies (7) $\geq$ (8). Note that equation (8) is specified so that investment in innovation $p_x\,x$ does not distort the bound of enforceable capital.[24]

The borrowing limit is increasing in financial wealth since the loss of collateral is greater in the case of default. It is also increasing in productivity and entrepreneurial ability, as only a share of output is kept in the case of default (see Amaral and Quintin, 2010; Buera et al., 2011).

5.7 Informal Sector Entrepreneurs

Informal sector entrepreneurs do not pay taxes but have no access to external finance. In addition, there is a sector specific marginal cost that is increasing in output, determined by parameter ξ. Profits for the informal sector firm are:

$$\pi(s,i) \;=\; q\,(1-\xi\,q) - (r+\delta)\,k - w\,l$$

The problem of the informal sector entrepreneur is:

$$v_i(s) \;=\; \max_{\{l,k,b'\geq 0\}} u(c) + \beta\,(1-\mu) \sum_{\{a'\}} \Lambda(a' \,|\, a)\, v(s') \tag{9}$$

$$\text{s.t.} \quad c + b' = \pi(s,i) + (1+r)\,b \quad \text{and} \quad k \,\leq\, b$$

and face the same occupational decision as workers (with $n = \underline{n}$):

$$v(s) \;=\; \max\{v_i(s), v_f(\varphi, \underline{n}, a, b - c_e), v_w(s)\}$$

The convex marginal cost specific to the production technology of informal sector firms makes it increasingly costly for larger firms to remain informal and is therefore a key determinant of the size of this sector and the size of firms in the sector. The literature has documented the worse access of informal sector firms to different types of public services and enforcement of property rights and the fact that informal sector firms are relatively small and unproductive.

[24] The following timing assumptions within a period imply that investment in innovation does not affect $\overline{k}(s,z)$: (1) entrepreneur observes shocks and rents capital, (2) production takes place, (3) capital is returned to the intermediary and financial assets are returned to the entrepreneur, (4) investment in the innovation good is decided. Innovation is financed with internal funds as it is subject to asymmetric information problems and cannot be easily collateralized (see Hall and Lerner, 2010; Gorodnichenko and Schnitzer, 2013).

5.8 Equilibrium

The state space is given by (φ, n, a, b, z), we previously defined $s = (\varphi, n, a, b)$ and $z \in \{i, f, w\}$. Given taxes and registration costs (τ, c_e) and interest rate r, a small-open economy stationary competitive equilibrium consists of:

- optimal quantities $\{q(s,z)\}_{z\in\{i,f\}}$, production inputs $\{l(s,z), k(s,z)\}_{z\in\{i,f\}}$,
- savings policy functions $\{b'(s,z)\}_{z\in\{i,f,w\}}$,
- policy function of investment in the innovation good $\{x(s,f)\}$,
- wage w, values $\{v(s), v_f(s), v_i(s), v_w(s)\}$, profits $\{\pi(s,z)\}_{z\in\{i,f\}}$,
- invariant measure $M(s,z)$ of individuals over the state space,

such that:

- workers solve (4), formal sector entrepreneurs solve (6), informal sectors entrepreneurs solve (9),
- market clearing condition in the labor market holds (entrepreneurs/managers plus workers equals the total mass of individuals), government revenues are dissipated,
- measure $M(s,z)$ is consistent with individuals' policy functions and optimal decision rules.

6 Baseline Parameters

The model parameters are divided into three groups: (1) a group of standard parameters taken from the literature, (2) a second group of parameters that are set to match key features of the US economy, (3) a group of country-specific and institutional parameters. Parameters in groups (1) and (2) are common for all countries in the model.

6.1 Common Parameters Across Countries

The interest rate r is set to 0.04 (Amaral and Quintin, 2010). The span-of-control parameter ν equal to 0.85 is taken from Atkeson and Kehoe (2005). The effective discount factor $\beta(1-\mu)$ of 0.92 is from Buera et al. (2011). We consider the case $\sigma \to 1$, log-preferences as in Midrigan and Xu (2013). Parameters α of 1/3 and δ equal to 0.08 are standard in the literature.

Table 4. Predetermined Parameters.		
parameter	value	description
$\beta(1-\mu)$	0.92	effective discount factor
σ	$\to 1$	risk aversion
r	0.04	interest rate (open economy)
ν	0.85	span-of-control
α	1/3	income share of capital
δ	0.08	capital depreciation rate
ρ	0.50	autocorrelation coefficient
σ_ε	0.40	standard deviation of shocks

For the parameters ρ and σ_ε that govern the idiosyncratic productivity process I take the mid-range of the values estimated by Abraham and White (2006) for a plant-level data-set that covers the manufacturing sector in the US for the period 1976-1999. The standard deviation is approximately equal to the median of the firm-level cross country estimates by Asker et al. (2012).

We now turn to the calibrated parameters in Table 5. The exogenous exit rate μ is set to match a total firm exit rate of 0.10. In the model the total exit rate equals the sum of the rate of entrepreneurs deciding to close their firms and the exogenous exit rate. Entrepreneurial ability is distributed according to a discrete Pareto distribution (truncated, with 15 possible values), its parameter is set to match the average size of firms in the US in the period 1995-2005 (Helfand et al., 2007).

Table 5. Calibrated Parameters - US Moments.		
parameter	par.	value
exogenous exit rate	μ	0.08
Pareto dist. (truncated, discrete, scaled)	$h(\varphi)$	0.72
innovation technology - level	ζ	25
innovation technology - curvature	λ	0.69
prob. *down* negative shock	ε	0.15
size innovation steps	Δ	0.36
target	target	model
firm exit rate	0.10	0.10
average firm size	22.2	22.0
average size age $20-24$/avg. size age <5	5.3	5.2
average size age $15-19$/avg. size age <5	4.1	3.9
average size age $10-14$/avg. size age <5	3.0	2.5
average size age $5-10$/avg. size age <5	2.0	1.4

The technology accumulation parameters target the life-cycle growth of firms in terms of labor as in Hsieh and Klenow (2012). For example, I target the average size of firms that are 15-19 years relative to firms that are younger than 5 years old for US manufacturing firms. With these parameters, the model underestimates the growth of firms, in particular at the earlier stages. Additionally, Midrigan and Xu (2013) find that for Korea (a developed economy), the ratio of total investment in intangibles over value added is 0.046 for a data-set of manufacturing firms. This value, however, is 0.01 in my model.

6.2 Country Specific and Institutional Parameters

Next, we need to specify parameters that are country specific or determined by institutions. The registration cost is from Djankov et al. (2002): it represents the cost of obtaining legal status to operate a firm, expressed as a share of per capita GDP in 1999. It includes all identifiable official expenses (fees, costs of procedures and forms, fiscal stamps, legal and notary charges, etc.) as well as the monetized value of the entrepreneur's time. The time of the entrepreneur is valued as the product of time required for registration and per capita GDP in 1999 expressed in per business day terms. Ignoring the time value component, the cost is 0.57 in terms of GDP per capita for Mexico and 0.15 for Colombia.

Table 6. Institutional/Country Specific Parameters.

description	par.	US	Mex.	Col.
total tax rate (% profits)*	τ	0.46	0.55	0.74
registration cost formal sector*	c_e	0.02	0.83	0.34
collateral constraint	ψ	1.00	0.25	0.34
informal sector convex cost	ξ	1.00	0.01	0.02
targets	**par.**	**US**	**Mex.**	**Col.**
private credit/output (formal sector)	ψ	2.3	0.2	0.2
% share of informal sector labor	ξ	0	46	49

*Source: World Bank and Djankov et al. (2002).
Reg. cost in terms of GDP per capita.

Parameter ψ determines financial development. As is standard in the literature, to set its value I target the ratio of private credit provided by financial institutions and private bond markets over GDP (Beck et al., 2009). For Colombia and Mexico the target corresponds to the middle of the period of the AMS-DANE dataset and for the formal sector following Midrigan and Xu (2013). The value for the US results in an economy with perfect financial markets (the average of the ratio for the 10 years between 1992-2001 is 2.3 which covers the period of the data used to impute firm life-cycle growth in Hsieh and Klenow, 2012).[25]

[25] Note that the amplification of misallocation refers to a comparison within a country keeping the level of financial development fixed and not a cross-country comparison. These exercises are discussed below.

The parameter that determines the convex marginal cost specific to the informal sector ξ affects the optimum production scale of informal sector firms. The target is the share of employment in the informal sector, equal to 0.45 for Mexico and 0.50 for Colombia. A lower value of ξ is necessary for Mexico relative to Colombia, since taxes are much higher in the latter case.

The tax rate τ, taken from the World Bank Doing Business Survey, is a measure of the total amount of taxes and mandatory contributions expressed as a share of commercial profits for a standardized business (after accounting for allowable deductions and exemptions). This measure considers taxes at all levels of government and includes the profit or corporate income tax, social security contributions, labor taxes paid by the employer and dividend taxes, among others. Taxes withheld (such as the personal income tax) or collected and remitted to tax authorities (such as value added taxes, sales taxes) are excluded. This measure simplifies a more complex tax structure that would distort capital labor ratios in the model.

7 Quantitative Analysis

In this section, the main quantitative results of the paper are presented and discussed.

7.1 Main Results

The main result of this paper is that misallocation losses in a model of financial constraints are amplified when we introduce endogenous firm-productivity accumulation. For exposition, we can equivalently define the potential gains from eliminating the dispersion across firms in the marginal productivity of capital. The focus is on the formal sector to avoid concerns related to measurement in the informal sector. Let J be the set of firms producing in the formal sector. It can be shown that TFP* in the case of no financial constraints is:[26]

$$TFP^{*} = \left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} \, (\varphi_j \, n_j) \right]^{1-\nu} \tag{10}$$

With financial constraints the marginal productivity of capital, and therefore the output-capital ratios, vary across firms and aggregate TFP is:

$$TFP = \frac{\left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} \, (\varphi_j \, n_j) \, (q_j/k_j)^{\frac{-\alpha\,\nu}{1-\nu}} \right]^{1-(1-\alpha)\,\nu}}{\left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} \, (\varphi_j \, n_j) \, (q_j/k_j)^{\frac{(1-\alpha)\,\nu-1}{1-\nu}} \right]^{\alpha\,\nu}} \tag{11}$$

An efficient allocation implies equalizing the marginal product of capital and therefore the average product as well. The gains from eliminating misallocation in

[26]See Midrigan and Xu (2013) and Buera et al. (2011).

the model are computed as $TFP^*/TFP-1$, with the following interpretation: keeping the set of firms and their productivity constant, this number represents the gains of eliminating differences in the marginal product of capital across firms. This exercise is analogous to the empirical studies in Hsieh and Klenow (2009) and Busso, Madrigal and Pages (2012).[27] Note that this is different from the comparison of aggregate TFP across countries, which I label the potential (total) TFP gain in Table 7. The latter comparison takes into account the fact that financial frictions also affect the distribution of firm productivity. For example, aggregate TFP is 19% lower in Mexico (formal sector) compared to the US.

Table 7. General Results.

variable*	**US**	**Mex.**	**Col.**
potential misallocation gain	–	14.7%	15.8%
potential (total) TFP gain	–	19.0%	23.4%
output per capita total	1.00	0.44	0.39
output per capita formal	1.00	0.47	0.42
output per capita informal	–	0.39	0.36
wage	1.00	0.47	0.43
capital/output total	2.34	0.58	0.56
total exit rate	0.10	0.13	0.13
total average firm size	22.0	6.3	5.8

*TFP and misallocation refer to the formal sector.

The results show that the potential misallocation gains are 14.7% and 15.8% for Mexico and Colombia in the model with endogenous firm productivity accumulation (Table 7). I also solve the model without knowledge capital accumulation, equivalent to setting $\Delta = 0$. It is not necessary to change the parameters of financial development ψ and taxes on profits τ. However, ξ needs to be modified keeping the same target of the size of the informal sector for each country, while c_e is changed to target its value relative to output per capita (neither one of these parameters enters TFP or TFP^* directly). Potential misallocation gains in this model are 7.3% and 3.8% for Mexico and Colombia, respectively. The larger amplification for Colombia reflects the higher level of taxes in that country.

Table 8. Misallocation: Knowledge Capital vs. No Knowledge Capital.

misallocation %	**Mex.**	**Col.**
knowledge capital	14.7	15.8
no knowledge capital	7.3	3.8

[27] In the model presented here, as in many models of financial frictions in the literature, the only source of dispersion in output-capital ratios is financial underdevelopment.

To decompose misallocation gains first define the following variables:

$$X = (e^a)^{1/1-\nu}\,(\varphi\, n) \qquad Y = \left(\frac{q}{k}\right)^{\frac{-\alpha\,\nu}{1-\nu}} \qquad Z = \left(\frac{q}{k}\right)^{\frac{(1-\alpha)\,\nu-1}{1-\nu}}$$

Without financial constraints, the unconstrained equivalents of Y and Z (derived from the first order conditions of the static profit maximization problem of the firm) are:

$$Y^* = \left(\frac{r+\delta}{\alpha\,\nu}\right)^{\frac{-\alpha\,\nu}{1-\nu}} \qquad Z^* = \left(\frac{r+\delta}{\alpha\,\nu}\right)^{\frac{(1-\alpha)\,\nu-1}{1-\nu}}$$

We can rewrite TFP in the model with financial constraints in the following manner:

$$TFP = J^{1-\nu}\frac{[\sigma(X,Y) + \mathbb{E}(X)\,\mathbb{E}(Y)]^{1-(1-\alpha)\,\nu}}{[\sigma(X,Z) + \mathbb{E}(X)\,\mathbb{E}(Z)]^{\alpha\,\nu}} \tag{12}$$

We can now decompose potential misallocation gains into two steps:

(1) Set Y and Z equal to its optimal unconstrained levels Y^* and Z^*. For Colombia, for example, this step generates a gain of 3.2% in the model without knowledge capital and 9.6% in the model with knowledge capital.

(2) Eliminate the covariances by setting $\sigma(X,Y) = \sigma(X,Z) = 0$. For Colombia, this step generates a gain of only 0.6% in the model without knowledge capital and 6.1% in the model with knowledge capital, given that $\sigma(X,Y)$ and $\sigma(X,Z)$ are more negative in the latter model. For Mexico, this step generates a gain of 2.1% in the model without knowledge capital and 6.9% in the model with knowledge capital.

The covariance terms reflect the fact that it is not only the variance in the marginal-productivity of capital that determines misallocation, but it is also important which firms are constrained. This is related to the discussion of the role of the correlation between firm productivity and distortions in Restuccia and Rogerson (2008) and Hopenhayn (2012). In the model without knowledge capital, due to the fact that shocks are mean-reverting, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is very constrained in one period can again be very constrained in the following period if the endogenous productivity component increases (this is further discussed below).

7.2 Firm Life-Cycle Productivity and Employment Growth

In Table 9 I compute the life-cycle growth and accumulation of knowledge capital for the three baseline model economies. By age 15, the ratio of $n\{\text{age=15}\}/\underline{n}$ is on average 15.7 in the US, but it is less than half this number for Colombia and Mexico. These differences in endogenous productivity accumulation translate into lower life-cycle growth of firms, as shown in the lower panel.

Table 9. Baseline Results: Firm Knowledge Capital and Size.

	USA	**Mexico**		**Colombia**	
knowledge cap.[1]	all	formal[2]	all	formal[2]	all
age 5/age 1	2.8	1.6	1.2	1.5	1.1
age 10/age 1	7.9	3.9	3.5	3.6	2.7
age 15/age 1	15.7	7.0	6.9	7.9	7.4
	USA	**Mexico**		**Colombia**	
# workers[3]	all	all		all	
age 6-10/age 1-5	1.4	1.2		1.2	
age 11-15/age 1-5	2.5	2.1		1.3	

[1]Average across firms of $n\{\text{age}=x\}/\underline{n}$.
[2]Firms that are formal at age x. [3]Includes manager.

Figures 3 and 4 show the model cross-section of $\log(n\,\varphi)$ including formal and informal sector firms with respect to age: the x-axis corresponds to the age of the firm and y-axis corresponds to $\log(n\,\varphi)$. I fit a quadratic polynomial to this relationship, where the number of simulated firms was increased until the results were unchanged. The range of $\log(n\,\varphi)$ incorporates an extensive-margin effect: in Mexico managers with lower entrepreneurial ability φ set up firms, specially in the informal sector (these firms are not included in the TFP/misallocation computations). The fitted value of $\log(\varphi\,n)$ is lower at every age in Mexico. To isolate the life-cycle component of knowledge capital, Figures 5 and 6 show the model cross-section of $\log(n)$ with respect to age for Mexico and US only for formal sector firms.

7.3 Firm Dynamics in the Model and Data

As previously discussed, the joint dynamics of output-capital ratios and firm productivity have implications for the impact of financial constraints on misallocation. In the model without knowledge capital, productivity shocks are purely stochastic and mean-reverting. In this case, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is constrained in one period can again be highly constrained in the following period if the endogenous productivity component increases. The table below shows that the model with knowledge capital is better able to replicate the dynamics between firm productivity and output-capital ratios estimated from the data.

Table 10. Regressions with Simulated Model and Data Firms.						
explained: **output/cap.**	**data**		**knowledge**		**standard**	
variables (logs)	(1)	(2)	(1)	(2)	(1)	(2)
lag - output/cap.*firm TFP	-0.02*	–	-0.04	–	-0.43	–
lag - firm TFP	–	-0.06*	–	-1.38	–	-3.38
lag - output/capital ratio	0.58*	0.58*	0.42	0.95	0.76	1.46
year-age controls, firm f. effects	yes	yes	yes	yes	yes	yes
industry controls	yes	yes	no	no	no	no
R^2 - within	0.37	0.37	0.49	0.52	0.56	0.64
Statistical significance *1%. (model regressions: all coefficients significant).						

In terms of firm productivity growth the model with knowledge capital also performs better than the standard model (Table 11).[28]

Table 11. Regressions with Simulated Model and Data Firms.						
explained: **TFP growth**	**data**		**knowledge**		**standard**	
variables[1] (logs)	(1)	(2)	(1)	(2)	(1)	(2)
firm TFP	-0.45*	-0.56*	-1.20	-1.10	-1.87	-1.75
output/capital ratio	0.05*	0.06*	0.27	0.22	0.40	0.34
age of firm	–	-0.03*	–	-0.10	–	-0.11
year controls, firm f. effects	yes	yes	yes	yes	yes	yes
industry controls	yes	yes	no	no	no	no
R^2 - within	0.19	0.24	0.33	0.34	0.44	0.45
Statistical significance *1%. (model regressions: all coefficients significant).						
[1]TFP growth computed between t and $t+1$, regressors in period t.						

7.4 Sensitivity Analysis

Introducing knowledge capital can affect the stochastic properties of firm productivity $e^a\,(\varphi\,n)^{1-\nu}$ and affect the level of misallocation. The role of persistence in firm productivity is discussed in Buera and Shin (2011). They argue that lower persistence may increase losses from misallocation. I compute the model without knowledge capital with a persistence parameter as low as 0.2 for Mexico: misallocation increases but only to 8.8% (increasing persistence reduces misallocation in my simulations). Although further exercises and alternative calibrations may be conducted, the stochastic properties of the process including knowledge capital do not

[28]This also holds if we consider firm productivity levels instead of firm productivity growth.

seem independently responsible for the increase in misallocation: in the baseline calibration of the model with knowledge capital for Mexico, the variance of the marginal productivity of capital is 0.15, close to the lower bound of 0.14 in Midrigan and Xu (2013) and slightly above the 0.12 for my model without knowledge capital.

8 Conclusions

The objective of this paper is to contribute to the understanding of the link between firm productivity dynamics and aggregate production efficiency. In particular I focus on TFP losses attributed to misallocation, which the empirical literature finds to be quantitatively important.

The underdevelopment of financial markets has been proposed as a source of misallocation. However, in a quantitative calibrated model, misallocation losses generated by financial underdevelopment are modest, as pointed out by Midrigan and Xu (2013). I find that considering a model with endogenous firm-productivity accumulation, the misallocation losses are amplified. In the case of Mexico financial constraints generate losses of 7.3% in a model without endogenous firm-productivity and 14.7% in a model with firm-productivity accumulation. This result suggests that the life-cycle accumulation in firm productivity can be important for understanding how financial constraints can generate misallocation. Furthermore, financial constraints affect the distribution of firm productivity and the level of aggregate TFP by distorting the accumulation of productivity at the firm level.

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A Output-Capital Ratios and Financial Constraints

Consider a standard profit maximization problem of a firm with access to a production technology with decreasing returns to scale (as the one in the quantitative framework) and productivity z:

$$\max_{\{k,l\}} \quad z\,(k^{\alpha}\,l^{1-\alpha})^{\nu} - w\,l - (r+\delta)\,k$$

subject to a restriction $k \leq \overline{k}$ (where $\overline{k}$ is derived from a financial constraint). It is straightforward to derive from the first order condition of capital that $\alpha\,\nu\,(q/k) = r + \delta + \gamma$ where $\gamma \geq 0$ is a multiplier on the financial constraint.

B Size Distribution of Establishments

The source of data for the US in Table A1 is Statistics of U.S. Businesses[29] (Census Bureau). Industries are classified according to NAICS:[30] manufacturing (codes 31-33), retail (44-45), services includes the following 2 digit categories: 48-49 (transportation and warehousing), 51 (information), 52 (finance and insurance), 53 (real estate and rental and leasing), 54 (professional, scientific and technical services), 55 (management of companies and enterprises), 56 (administrative and support and waste management and remediation services), 71 (arts, entertainment and recreation), 72 (accommodation and food services), 81 (other services, except public administration). I exclude the following categories: 61 (educational services), 62 (health care and social assistance), 92 (public administration), 11 (agriculture, forestry, fishing, hunting), 21 (mining, quarrying, oil and gas extraction), 22 (utilities), 23 (construction). The group *all sectors* includes the total of the non-excluded categories, this accounts for a total of 78.5 million workers. Including all the categories accounts for 111.97 million workers (the size distribution including all categories does not show significant changes).

For Mexico, the data in Table A1 is from Busso, Fazio and Levy (2012), corresponding to the Economic Census 2004. We note that the data understates true differences given the under-representation of small firms in Mexico, in particular those in the informal sector (see Busso, Madrigal and Pages, 2012; Leal Ordoñez, 2013).

[29] http://www.census.gov/econ/susb/
[30] http://www.census.gov/cgi-bin/sssd/naics/naicsrch?chart=2007

Table A1. Distribution of Establishments and Employment.

USA[1]	**establishments**				**employment**			
shares in %	<5	5-9	10-49	≥50	<5	5-9	10-49	≥50
all sectors	47.52	12.56	⇒	39.92	5.32	5.48	⇒	89.20
manufacturing	36.81	15.77	⇒	47.43	1.89	2.90	⇒	95.21
manufacturing[2]	40.47	14.05	29.21	16.27	1.90	2.28	15.77	80.06
retail	37.85	12.67	⇒	49.47	5.18	5.97	⇒	88.85
services	50.96	12.29	⇒	36.76	6.06	5.87	⇒	88.06
Mexico[3]	**establishments**				**employment**			
shares in %	≤5	6-10	11-50	>50	≤5	6-10	11-50	>50
all sectors	90.62	4.95	3.49	0.94	35.17	7.50	14.86	42.47
manufacturing	83.42	7.44	6.01	3.13	13.85	4.30	10.28	71.56
retail	93.37	3.67	2.47	0.49	53.58	8.47	15.76	22.19
services	88.64	6.14	4.28	0.95	34.67	9.23	17.83	38.26

[1]Statistics of U.S. Businesses (CB), [2]Cole, Greenwood and Sanchez (2012), [3]Busso, Fazio and Levy (2012).

C Dispersion in Output-Capital Ratios in Colombia

Table A2 documents the dispersion in the log-output/capital ratios for the AMS database, where capital includes buildings, structures, machinery and equipment. In each year and for each 4 digit industry I take firms with more than 10 workers. Within each industry that has more than 20 observations I compute the variance and the difference in the levels of different percentiles. For each year I then take the average of a particular measure across industries.

Table A2. Dispersion in Output-Capital Ratios.

statistic[1]	1982	1986	1990	1994	1998	82-98[2]
variance	1.7	1.6	1.6	1.7	2.2	1.7
90-10 percentiles	3.1	3.0	3.0	3.2	3.5	3.1
85-15 percentiles	2.4	2.3	2.4	2.6	2.8	2.5
80-20 percentiles	2.0	1.9	1.9	2.0	2.2	2.0
80-50 percentiles	0.9	1.0	1.0	1.0	1.1	1.0
# 4 digit industries	57	60	59	57	54	57
avg. # observations per industry	83	81	84	70	68	78

Source: computed w/AMS-DANE (1982-1998).
[1]Average across 4 digit industries, ratios in logs. [2]Average 17 years.

D Micro-Enterprizes in Mexico

The National Survey of Micro-Enterprizes (ENAMIN) is conducted every two years and includes data on firms with up to 15 workers in manufacturing, and up to 10 workers in construction, transportation, retail and services. INEGI estimates that approximately 41.6% of the labor force belongs to firms in this scale of production (approximately 18.1 million workers). The data collected by this survey includes information on the manager/owner of the firm: education, experience, time in present position and reasons for setting up a business, among other variables. Regarding the firm itself, the information collected includes: year the business was established, accounting and registry, equipment, expenditures, investment, income, access to finance and number of workers, among other variables.

The survey provides information of registration of the firm (whether it belongs to the informal sector). Approximately 17.6 thousand of the business managers/owners replied that they had not initiated any formal process of registration with Government authorities in the 2010 survey (6.9 thousand replied that they had). Considering the firms that had not initiated any formal registration process the main activities (accounting for 70% of the group of non-registered firms) were the following: retail of food, beverages and tobacco (code 4611, 11.6%), preparation of food and beverages (7221, 8.6%), intermediation and retail of massive communications media (4690, 7.3%), contractors in construction (2382, 5.7%), food industry (3110, 5.5%), personal services (8121, 4.1%), preparation of food and beverages without fixed location (7222, 3.8%), repairs and maintenance of equipment, machinery, household and personal appliances (8112, 3.6%), retail of food and beverages without fixed location (4612, 3.5%), maintenance of automobiles and trucks (8111, 3.3%), fabrication of clothing (3150, 3%), residential construction (2361, 2.9%), retail of clothing, accessories and footwear (4631, 2.9%), retail of clothing, accessories and footwear without fixed location (4632, 2.4%), manufacturing of textiles except clothing (3140, 2.3%).

Fig. 1. Employment Growth over Life Cycle − Model

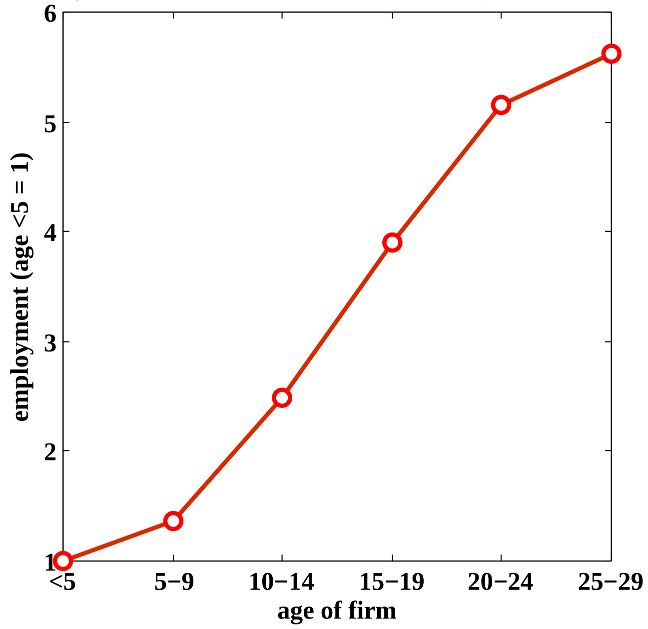

Fig. 2. Occupation Choice (Mexico/Colombia)

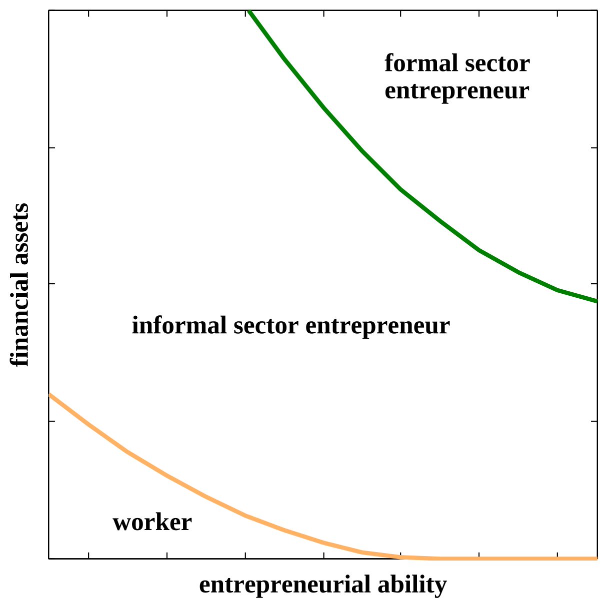

Fig. 3. log(ability*knowledge) − Mexico

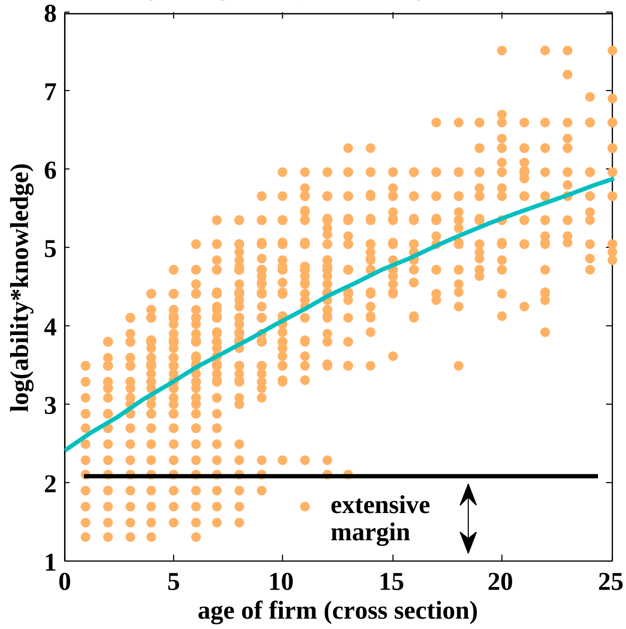

Fig. 4. log(ability*knowledge) − US

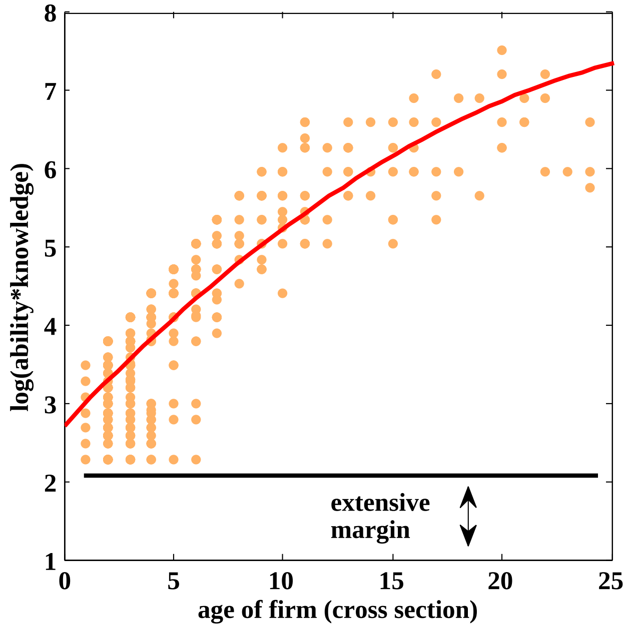

Fig. 5. log(knowledge) - Mexico (formal sector)

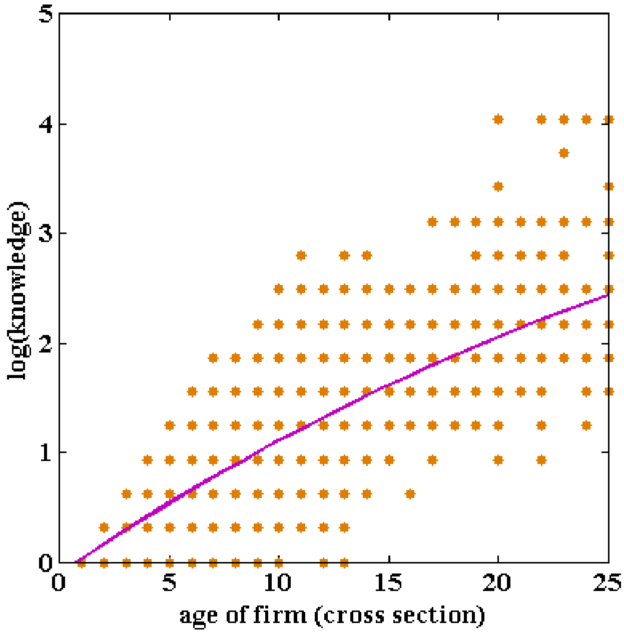

Fig. 6. log(knowledge) - US (formal sector)