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**INTERPRETATION OF POPULATION DENSITY GRADIENTS:  
A BRAZILIAN PERSPECTIVE**

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## **I – Introduction**

One thread of quantitative (and graphic) studies of the spatial form of metropolitan areas seeks to relate the variation of metropolitan urban form in time and space to its determinants, particularly, physical topography, large-scale transportation investments, income distribution, stage of urban development and regulatory regime. An important and traditional variant of this line of research traces metropolitan density gradients and interprets their variation in cross-section and time series samples according to theorems and hypotheses suggested by urban economic theory<sup>1</sup>. Dependent variables of the urban gradients studied include population, employment (total and sectoral), built density, income, land-value, and building height.

Among the instruments developed by the urban economics theory, the negative exponential function of the Population Density Gradient – PDG has proved to be useful in measuring the form of the distribution of population in urban areas. This instrument was originally proposed by Colin Clark's "Urban Population Densities", *Journal of the Royal Statistical Society*, 1951, cited by Mills as "the first extensive study of demographic density functions". A large number of studies have been produced by using the PDG to measure the degree of decentralization of population in cities worldwide as well as the evolution of urban sprawl in individual cities. Although largely applied in academic spheres, the density gradient function is not commonly used to support the formulation of public policies, particularly in Brazil.

As demonstrated by literature, the common (but not exclusive) pattern of the PDG in cities is its negative slope with distance from the city center. Central areas are densely populated while in the peripheries the density reaches low values. In this paper we present estimation results of the Density Gradient of Population for ten of the largest Brazilian cities: Fortaleza, Recife, Salvador, São Paulo, Rio de Janeiro, Curitiba, Porto Alegre, Belo Horizonte, Goiânia and Brasília.

All these Brazilian cities experienced fast urban growth during the 20th century. They are state capitals and urban centers of important metropolitan areas which experienced different paces of urban growth. São Paulo and Rio de Janeiro had their spatial structure defined by the

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<sup>1</sup> The original works of Alonso, Muth and Mills and recent extensions of that body of analysis, for example, by Glaeser and Malpezzi-Bertaud.

decentralized mechanisms of choice. This is a characteristic of open economies due to the absence of strong land use regulation, at least until its basic structure is defined. In fact, São Paulo and Rio de Janeiro are particularly important because they are the largest cities in Brazil as well as central cities of the main metropolitan areas of the country.

Fortaleza, Recife, Salvador, Curitiba and Porto Alegre are examples of large regional centers developed during the period of urbanization of the second half of the 20th century. Fortaleza and Recife are important cities of the northeast, the poorest region of Brazil. Their economy traditionally relies on agriculture, but not exclusively so. Salvador, also in the northeast, has a set of industries linked to the petroleum industry which was developed in the last 20 years. Curitiba and Porto Alegre are important regional centers of the southern region which is richer and more industrialized than the northeast.

Located in the central-west, Belo Horizonte was built to be the new capital of the state of Minas Gerais in 1897, following an urban plan that pre-defined the land use of its areas. Similarly Goiânia, inaugurated in 1942, also located in the central-west, was planned to be the capital of Goiás. Brasília, yet another planned city, was inaugurated in 1960 to be the capital of Brazil followed a master plan which applied the principles of the functional city as advocated by the modern movement of architecture. Since its inauguration, the city administration adopted regulatory instruments that imposed strong constraints on land use in order to achieve environmental and urbanistic objectives.

This study investigates the spatial pattern of population distribution in the cities mentioned and the effect of regulation and regulatory regimes on urban spatial form. We use Brasília as an example of a city with strong land use regulation and compare its structural distribution of population with the patterns found in the other cities. We found that, with the exception of Brasília, the other nine cities of our sample presented a similar urban distribution of population despite their different origins and urban growth processes. Population Density Gradient of those cities is negatively sloped, as predicted by the urban economy theory and generally found in most cities worldwide. Brasília, on the other hand has a Density Gradient that is practically flat.

Actually, if we consider only the legalized urban areas of Brasília, that is, the city built by the settlements promoted by administrative decisions in the absence of a free private land market

and following the land use prescriptions, the density gradient is positively sloped. In Brasilia, the densest areas are located far from the city center, a part of *Plano Piloto* registered as Cultural Heritage Site of the World by UNESCO. As a result, the *Plano Piloto* is object of strong constraints on changing its original plan. The location of most settlements far from the central areas produced a median distance per person to the city center of about 20 kilometers, the longest average found in our sample.

The Population Density Gradient has a slight negative slope if we include in the calculation the informal settlements formed by the decentralized market decisions. That is an important result that can be emblematic of the importance of free land markets in efficiently land allocation.

## **II – The Urbanization of Brazilian cities**

The urbanization process in Brazil is quite recent, occurring mainly during the second half of the 20th century. While in 1950 about 64% of the people in Brazil were living in the rural areas, in 2000 this pattern was inverted and 81% of the Brazilian population was living in urban areas. During these fifty years, the urbanization of the country can be divided basically in two periods. The first, marked by the industrialization of the country, starts in the thirties and extends to the seventies and is characterized by the concentration of population in the main state capitals located in the southeast region. This process created great metropolitan areas surrounding cities such as São Paulo, Rio de Janeiro and Belo Horizonte that polarized the migration flows from the whole country. The second period goes from the seventies to the present days and is characterized by the generalization of the urbanization process across the country. During this period many cities developed as important regional centers, mainly in the capitals of the west-central, south and northeast regions.

Historically, the development of Brazil was concentrated in the coast, leaving the countryside scarcely populated. Although most of the country population lived in the hinterland, its vast interior maintained low gross densities and showed large, virtually uninhabited areas. As a consequence, the major cities of Brazil and the denser areas are still located along the coast, mainly in the southeast, south and northeast regions. Between 1940 and 1980 the urban population of Brazil grew at a rate of 4.8% by year, while the urban population in those regions

grew at rates of 4.7%, 5.2% and 4.2%, respectively. Even the central-west region, which had 2.1% of the urban population of Brazil in 1940, saw its urban population grow 7.6% by year in the same period.

In this the period, Brazil changed from rural to urban country, and the main metropolitan areas of São Paulo, Rio de Janeiro and Belo Horizonte arose supported by industrialization of these cities. By 1970, more than half of the country's population (55.9%) was living in urban areas, a share that was captured by the census data shown in Table 1. In that year the southeast region was the only one that had more people in cities (72.7%) than in the countryside.

**Table 1 - Urbanization rates for Brazil and its regions, 1940-2000**

<b>Brazil and regions</b>	<b>1940</b>	<b>1950</b>	<b>1960</b>	<b>1970</b>	<b>1980</b>	<b>1991</b>	<b>1996</b>	<b>2000</b>
<b>Brazil</b>	31.2%	36.2%	44.7%	55.9%	67.6%	75.6%	78.4%	81.2%
<b>North</b>	27.7%	31.5%	37.4%	45.1%	51.6%	59.0%	62.4%	69.9%
<b>Northeast</b>	23.4%	26.4%	33.9%	41.8%	50.5%	60.7%	65.2%	69.1%
<b>Southeast</b>	39.4%	47.5%	57.0%	72.7%	82.8%	88.0%	89.3%	90.5%
<b>South</b>	27.7%	29.5%	37.1%	44.3%	62.4%	74.1%	77.2%	80.9%
<b>Central-West</b>	21.5%	24.4%	34.2%	48.1%	67.8%	81.3%	84.4%	86.7%

Source: Demographic Census, IBGE.

The industrialization was intense in some of the major cities of Brazil, particularly São Paulo. This process was partly supported by government incentives such as the policy of import substitution and the protection of the domestic market from international competition. São Paulo and Rio de Janeiro had the primacy in attracting industries, not only due to political interests but also due to the early industrial infra-structure and significant labor and consumer markets that these cities provided. In addition, the growth of urban labor markets in these cities encouraged mass migration from near states and distant northeastern rural areas where the climate and the low productivity made life virtually unsustainable.

Hence, the urban growth of the great metropolitan areas of the southeast region formed until the 1970s was constituted most of poor people from the depressed areas of northeast that suffered from long periods of drought. As shown in Table 2, the relative participation of the southeast region in the urban population in Brazil is above 50% until 1980. However it decreases to 49.8% in 1991, and 47.8% in 2000, as result of the generalized urban growth in cities located

in other regions. Nevertheless, the southeast region still concentrates most Brazilian urban population.

**Table 2 - Relative participation of the regions in the Brazilian urban population 1940-2000**

Region	1940	1950	1960	1970	1980	1991	1996	2000
<b>North</b>	3.2%	3.1%	3.1%	3.1%	3.8%	5.3%	5.7%	6.5%
<b>Northeast</b>	26.3%	25.3%	24.0%	22.6%	21.8%	23.2%	23.7%	23.9%
<b>Southeast</b>	56.1%	57.1%	55.8%	55.6%	53.3%	49.8%	48.6%	47.5%
<b>South</b>	12.3%	12.3%	13.9%	14.0%	14.8%	14.8%	14.8%	14.7%
<b>Central-West</b>	2.1%	2.3%	3.2%	4.7%	6.4%	6.9%	7.2%	7.3%

Source: Demographic Census, IBGE.

Cities in other regions also grew significantly. In the period from 1940 to 1970 the regions northeast and south, for example, showed urban growth rates of 4.2% and 5.2% per year, respectively, affecting particularly cities like Salvador (Bahia) and Recife (Pernambuco) in the northeast and Porto Alegre (Rio Grande do Sul) and Curitiba (Paraná) in the south, for example.

The share in the Brazilian urban population of the central-west region also increased in the same period, rising from 2.1% in 1940 to 4.7% in 1970. The central-west was the region that showed the highest urban growth rates, 7.6% per year between 1940 and 1970, despite its small share in the total urban population of Brazil. Here it is important to distinguish the role that the new cities like Belo Horizonte, Goiânia and Brasília represented in the urbanization of the interior. Belo Horizonte was built to be the capital of the state of Minas Gerais, and later Goiânia was built to be the capital of the state of Goiás. Finally, Brasília was built to be the capital of the country. These three cities, especially Belo Horizonte and Brasília, played an important role in increasing occupation of the hinterland of the country.

The trend of concentration of urbanization in the cities located in the coast starts to change from the 1970s. The process of urbanization becomes more generalized and the number of cities increased suggesting changes in the Brazilian urban network. In that decade the urban network of Brazil becomes more complex with the consolidation of the metropolitan areas around the main cities of the earlier period as well as the development of some urban centers in the hinterland that acquired regional and local importance. Nowadays, the growth of the metropolitan nuclear cities is still positive, but at declining rates.

One aspect of the declining growth of metropolitan areas is related to the cities that became important regional centers, such as the cities of the central-west, north and the northeast regions. Salvador, Goiânia, Manaus and Fortaleza are examples of cities that have presented high urban growth rates in the last 20 years, attracting the migration flow that traditionally would have been directed toward the great cities of the southeast.

One of the reasons for this change was the decentralization of urban employment. For the last 20 years many industries have left the large urban centers of the southeast because of high production costs due to diseconomies that generally affect great metropolitan areas. Many times the transference of the industries was stimulated by incentives offered by other states, in the shape of lower tax policies (known as the "Fiscal War") and cheaper labor force.

The development of many urban centers forming new consumer markets encouraged some industries located in great metropolitan areas to move to cities that offered easier access to major regional market shares. New urban nodes of goods and services distribution developed, linking the metropolitan areas to the cities that played a regional role. Within the national market these cities fulfill more traditional urban functions, while the metropolitan areas become more specialized.

The urban scenario in Brazil at the end of the century presented 81% of its population living in cities. In terms of the regions, the southeast had in 2000 a share of 90.5% of its population living in cities, followed by the central-west where 86.7% of its population lived in urban areas. The north and northeast are the regions that have the smallest proportion of its population living in cities, both with less than 70% in 2000.

Despite the generalized urbanization process experienced by Brazil for the last thirty years, the urban population of the country is still predominantly distributed along the coastline, even though a large number of cities located in the hinterland are now playing an important regional and local role. At the same time some metropolitan areas have reduced their influence over space and some, such as Rio de Janeiro, are losing their force at the national level. In contrast Belo Horizonte continues to enlarge its influence while new cities, such as Goiânia, acquire regional importance.

The panorama presented in this section offers a macro view of the development of Brazilian cities during the second half of the 20th century. In general, the internal spatial

organization of cities reflects some of the circumstances of this process, which associated with some particular conditions, yield different spatial structures. One of the features shared by these cities is the pattern of distribution of population in urban areas. Generally, big cities in Brazil present a pattern of population density that decreases from their central areas toward their outer limits. In addition, the rapid growth experienced by the Brazilian cities was absorbed in their peripheral areas, what is reflected in the declining population density gradient measured along time.

### **III – Population Density Gradients for 10 Brazilian cities**

Generally, Brazilian cities present a pattern of growth that is similar to many cities worldwide, not only in industrialized, but also in developing countries. As such, with rare exceptions, they grew around a single center where most of commerce and employment was concentrated. This pattern, which one might call "monocentric polarization", fits most models presented by scholars to explain the way that cities grow and form. Best known is that model developed by Alonso (1964), Muth (1969) and Mills (1972) that analyzes urban form combining micro-economics principles in a Von-Thünen spatial arrangement. This model is characterized by central polarization and considers urban land value and commuting costs as a function of the distance to the city center.

This model and the variants derived from it – like the models that analyze subcenters formation in metropolitan areas –, assume a decentralized decision process where the location choices are taken by private agents –developers, firms and dwellers – that compete and bid for urban locations. In equilibrium the land is allocated efficiently to whoever is willing to pay more for the most profitable sites of the city. In this process, land prices exert pressure on the whole city and tend to be higher in the more accessible sites of the urban area, like the city center and along the main transport corridors. In the farthest areas from the city center and less accessible areas land prices tend to be lower. Basically, when agents bid for the best urban locations they trade-off land price and commuting costs across the whole city, in order to choose between the many possible locations for their activities. That process is the principle of the efficient allocation of land in cities within a free land market.

High land prices at the more accessible sites induce factor substitution that decreases the quantity of land employed per unit of floor built area. In other words, the high land prices in central areas and along the transport corridors encourage developers to substitute no land inputs (capital, labor, construction materials) for land. As such, in the case of housing production, developers will use less land per unit of housing where the land price is higher.

This substitution effect induces the rise in the ratio of total housing floor area to total land area (called floor area ratio, FAR) producing higher densities where the land prices are higher. That is how central locations generate higher densities while lower densities occur on the fringes of the city. The decrease of density with distance from the city center produces a pattern found in many cities worldwide, nowadays and over the course of urban history.

By using this theoretical approach (developed by Alonso, Muth, Mills – AMM) it is possible to interpret the variation of population density with distance from the city center by estimating the parameters of the negative exponential function shown below, using simple regression methods.

$$D(\mu) = d_0 \cdot e^{-\gamma\mu} \quad (1)$$

The dependent variable of the model is a vector formed by all densities  $D$  observed at the many city locations as function of distance to the city center ( $\mu$ ), and the estimated parameter ( $-\gamma$ ) is called Population Density Gradient. The parameter  $d_0$  is the constant estimated in the regression calculation and represents the density estimated for the center of the city. The Density Gradient is useful not only to measure the distribution of the population across the urban area, but also to measure the increase or decrease of urban sprawl over time.

We used this model to estimate the Population Density Gradient for ten of the largest capitals of Brazil. The chosen cities are located in four most urbanized regions of Brazil, the northeast, southeast, south and central-west and represent the urbanization processes described in the prior section of this paper.

### ***Brazilian cities and their urban density pattern***

The ten Brazilian cities chosen for this study, with the exception of Brasília, have had similar patterns of growth in spite of the different circumstances of their origin and development. The chosen cities are Rio de Janeiro, São Paulo and Belo Horizonte, located in the southeast region, the most urbanized and industrialized region in the country; Porto Alegre and Curitiba, located in the south; Fortaleza, Recife and Salvador, located in the northeast, the poorest region of the country, and Goiânia and Brasília, located in the central west. Brasília, built to be the capital of Brazil and inaugurated in 1960 will be analyzed separately due to some specific details of its urban structure and development that make it of special interest to our study. Together these ten cities Table 4 shows population in the years of 1940, 1970 and 2000 for the cities cited above, and the respective annual growth rates for the periods between these years.

Rio de Janeiro and São Paulo are the largest Brazilian cities, as well as the most industrialized. Their growth during the 20th century, especially in its first half, was supported by intense industrialization concentrated in their urban core and surrounding metropolitan area. Together, these two cities had almost 20% of the whole urban population of Brazil in 1970. This degree of urban concentration declined to a little more than 11% by the end of century. The development of their urban areas took place in the presence of private land markets, generally unconstrained by strong land use regulations. Their spatial structure was formed by the addition of new developed rings around their central areas, following the classic pattern of a concentric growth. Even in the city of the Rio de Janeiro, where the amenities of the littoral can weaken the attractiveness of the central area and topography offers some geographic constraints, the spatial structure follows the predictions of the theory.

It is appropriate to mention here that, despite some plans like the Avenue Plan of Prestes Maia for São Paulo in 1930 and some plans for the Rio de Janeiro, like the *Plano Agache* in the 1920s as well as others, in the period of intense urbanization there was practically no effective regulation of urban land use. The plans implemented, with rare exceptions, were principally concerned with remodeling and embellishing central areas of the cities. Hence, not only Rio de Janeiro and São Paulo, but most Brazilian cities grew in a relatively permissive environment. Zoning and other tools of land use regulation were more widely adopted in the sixties and seventies when SERFHAU (the Federal Service of Housing and Urbanism) was created and

structured to deal with urban planning across the country. SERFHAU was not only concerned with physical aspects of urban planning but also sought to integrate social and economic aspects into urban planning.

**Table 4 - Total Population and Growth Rates for Selected Brazilian Cities**

City		Year		
		1940	1970	2000
<b>São Paulo</b>	Population	1,326,261	5,924,612	10,434,252
	Annual Growth Rates		<b>5.12%</b>	<b>1.90%</b>
<b>Rio de Janeiro</b>	Population	1,764,141	4,251,918	5,857,904
	Annual Growth Rates		<b>2.98%</b>	<b>1.07%</b>
<b>Fortaleza</b>	Population	180,185	857,980	2,141,402
	Annual Growth Rates		<b>5.34%</b>	<b>3.10%</b>
<b>Recife</b>	Population	348,424	1,060,701	1,422,905
	Annual Growth Rates		<b>3.78%</b>	<b>0.98%</b>
<b>Salvador</b>	Population	290,443	1,007,195	2,442,107
	Annual Growth Rates		<b>4.23%</b>	<b>3.00%</b>
<b>Curitiba</b>	Population	140,656	609,026	1,587,315
	Annual Growth Rates		<b>5.01%</b>	<b>3.24%</b>
<b>Porto Alegre</b>	Population	272,232	885,545	1,360,590
	Annual Growth Rates		<b>4.01%</b>	<b>1.44%</b>
<b>Belo Horizonte</b>	Population	211,377	1,235,030	2,238,526
	Annual Growth Rates		<b>6.06%</b>	<b>2.00%</b>
<b>Goiânia</b>	Population	48,166	380,773	1,093,007
	Annual Growth Rates		<b>7.13%</b>	<b>3.58%</b>
<b>Brasília</b>	Population	-	537,492	2,051,146
	Annual Growth Rates			<b>4.57%</b>

Source: Authors calculations using the 1991 and 2000 Census data, IBGE.

SERFHAU developed plans for hundreds of Brazilian cities, but while these included land use regulations, only a few were effectively implemented. From then on the creation of instruments for urban land regulation to control the urban sprawl became more common in Brazilian cities. In 1979, a federal law introduced norms for the development of urban areas

across the country. This law, with some changes, is still in force despite the Federal Constitution of 1980 delegating urban land use regulations as a prerogative of local governments.

The cities of Salvador, Recife and Fortaleza, in the northeast, and Porto Alegre and Curitiba, in the south, are example of cities that also experienced intense urban growth, with few restrictions on land occupation and in the presence of an unregulated land market. Recife and Porto Alegre experienced high growth rates, between 3.8% and 4% per year, respectively until the 1970s. These rates declined to 1.44% and 0.98%, respectively, between 1970 and the end of last century. Salvador, Fortaleza and Curitiba maintained high growth rates, above 3% per year until the end of last century. Generally, the spatial structure of these cities follows the classic model that predicts high densities in the central areas and low densities at the outer limits of the city. We observe that the geography of Recife, due to the canals that divide its territory, especially near the city center, tends to reduce central densities and flatten the Density Gradient.

The cities of Belo Horizonte and Goiânia share a singular characteristic: they were designed and built to be state capitals, the first in 1897 and the second in 1942. Similarly, Brasília was designed to be the nation's capital and was inaugurated in 1960. The numbers in Table 4 show that Belo Horizonte and Goiânia were the cities that had the most intense growth until 1970, while Brasilia's growth was outstanding in the second period. Indeed, currently Belo Horizonte is the fourth largest city in Brazil and the most industrialized city in the interior, despites belonging to the southeast region. Goiânia and Brasília are the largest cities in the central west, indicating that the project of stimulating the occupation of the hinterland building cities was successful. Furthermore, Brasília is the sixth city of Brazil which, in spite of its age, has overtaken older and more industrialized cities as Recife, Curitiba and Porto Alegre.

Differently than other cities, Belo Horizonte, Goiânia and Brasília were built in response to political objectives and did not owe their initial appearance to comparative advantage, like commercial cities. They were originally built to be administrative centers, even though Belo Horizonte developed a significant industrial base. The economy of Goiânia and Brasília is supported by commerce and services, but Brasilia's economy relies mainly on government services.

The location and spatial arrangements of these cities were predetermined before their birth. Hence, the spatial structure of these cities had already been decided before land market

mechanisms established land values and the distribution of economic activities and population across the urban area. That is in itself neither good nor bad, but the specific institutional and urban context created in the city can establish special conditions for the land market. In unregulated markets any misallocation of land due to any failure of the project or unexpected transformation of the general conditions of the society can be adjusted through the decentralized mechanism of land market. But strong and rigorously implemented urban regulations that constrain land use can obstruct the recycling of older areas and perpetuate inefficient land allocation occasionally yielding greater costs than benefits.

Table 5 presents the results of the estimation of Population Density Gradients for the ten cities analyzed in two moments: 1991 and 2000 (except Brasília for which we obtained data only for 2000). The regressions employ IBGE census data for 1991 and 2000 for all the cities. The territorial unit considered for data aggregation was, for most cities, neighborhoods, districts for the city of São Paulo and information zones for Salvador. Brasilia was a special case since its urban structure is extremely fragmented and there is no district or neighborhood subdivision. We selected areas where land use and occupation were homogeneous according to data from its several census sectors. While Brasilia is actually divided in 26 Administrative Regions, including urban and rural areas in each region, we selected 62 areas where urban occupation is predominant, purging rural areas.

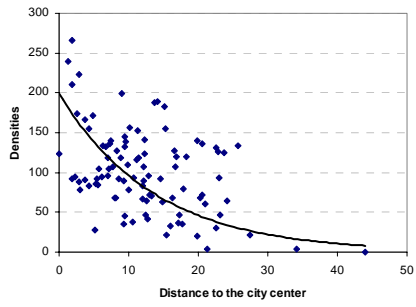
All ten cities presented negative density gradients representing decreasing densities when approaching the outer limits of the cities. In addition, for nine cities we had data for 1991 and 2000 that showed the value of the Density Gradients decrease in the period. This indicated that urban growth was decentralized and tended to be concentrated in peripheral areas during the last decade of last century. This decentralization of population was also tied to a decrease in population density in the central areas (except in Fortaleza and Curitiba). This is supported by cumulative population within a 3 km radius around the CBD as well as the decrease of the regressions constants during the period studied. Indeed, the reduction of estimated regression constants for density in all of the cities studied reflected a population reduction within the city center ( $d_0$  in equation 1). Figure 1 shows the plots of the Population Density Gradients for the ten cities analyzed.

**Table 5 - Densities and Population Density Gradients for 10 Brazilian Cities in 1991 and 2000**

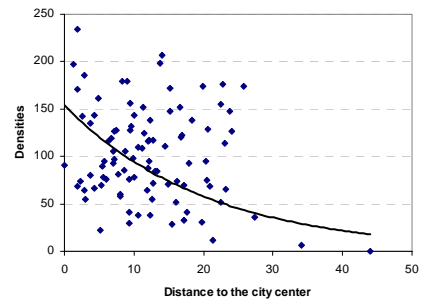
City	Area (km <sup>2</sup> )	Gross Density 2000 (people/ha)	Population 2000	Average distance per person to city center (km)	Population Density Gradient 1991			Population Density Gradient 2000		
					Constant	Density Gradient	Adjusted R <sup>2</sup>	Constant	Density Gradient	Adjusted R <sup>2</sup>
<b>São Paulo</b> <i>(Prob  T  &gt; t)</i>	1,525	68.4	10,434,252	14.1	5.30 <i>(0,0000)</i>	-0.073 <i>(0,0000)</i>	35.7	5.04 <i>(0,0000)</i>	-0.049 <i>(0,0000)</i>	19.8
<b>Rio de Janeiro</b> <i>(Prob  T  &gt; t)</i>	1,225	78.0	5,857,904	19.9	5.13 <i>(0,0000)</i>	-0.040 <i>(0,0000)</i>	24.5	5.00 <i>(0,0000)</i>	-0.029 <i>(0,0000)</i>	15.5
<b>Fortaleza</b> <i>(Prob  T  &gt; t)</i>	330	64.9	2,141,402	8.0	5.33 <i>(0,0000)</i>	-0.166 <i>(0,0000)</i>	27.6	5.14 <i>(0,0000)</i>	-0.108 <i>(0,0000)</i>	17.9
<b>Recife</b> <i>(Prob  T  &gt; t)</i>	218	65.3	1,422,905	6.8	4.89 <i>(0,0000)</i>	-0.098 <i>(0,0196)</i>	4.8	4.80 <i>(0,0000)</i>	-0.071 <i>(0,0767)</i>	2.3
<b>Salvador</b> <i>(Prob  T  &gt; t)</i>	279	87.4	2,436,390	8.1	5.39 <i>(0,0000)</i>	-0.146 <i>(0,0000)</i>	26.7	5.29 <i>(0,0000)</i>	-0.100 <i>(0,0000)</i>	19.4
<b>Curitiba</b> <i>(Prob  T  &gt; t)</i>	435	36.5	1,587,315	7.6	4.52 <i>(0,0000)</i>	-0.191 <i>(0,0000)</i>	52.7	4.32 <i>(0,0000)</i>	-0.134 <i>(0,0000)</i>	34.7
<b>Porto Alegre</b> <i>(Prob  T  &gt; t)</i>	375	28.8	1,360,590	7.7	5.11 <i>(0,0000)</i>	-0.187 <i>(0,0000)</i>	40.7	5.06 <i>(0,0000)</i>	-0.168 <i>(0,0000)</i>	42.0
<b>Belo Horizonte</b> <i>(Prob  T  &gt; t)</i>	331	67.6	2,238,526	7.2	4.80 <i>(0,0000)</i>	-0.082 <i>(0,0017)</i>	12.0	4.73 <i>(0,0000)</i>	-0.052 <i>(0,0211)</i>	6.0
<b>Goiânia</b> <i>(Prob  T  &gt; t)</i>	385	28.6	1,093,007	6.7	4.39 <i>(0,0000)</i>	-0.203 <i>(0,0000)</i>	32.8	3.97 <i>(0,0000)</i>	-0.150 <i>(0,0013)</i>	11.8
<b>Brasília</b> <i>(Prob  T  &gt; t)</i>	5,822	3.5	2,051,146	20.1	-	-		3.09 <i>(0,0000)</i>	-0.003 <i>(0,0000)</i>	<i>(R<sup>2</sup> = 0.0)</i>

Source: Census data and the authors' calculations.

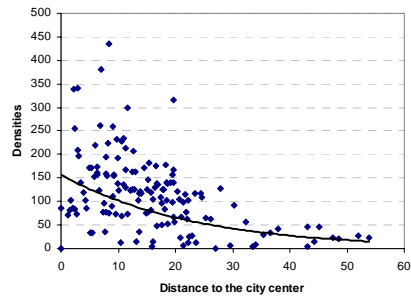
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**SÃO PAULO 2000**



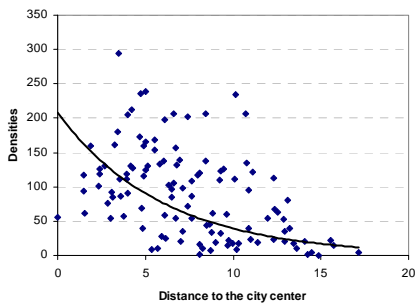
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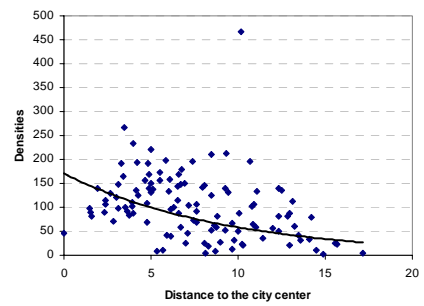
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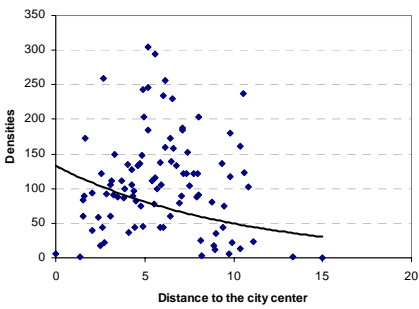
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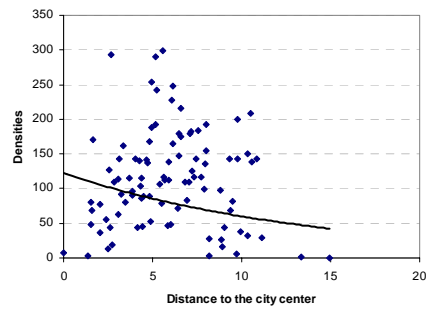
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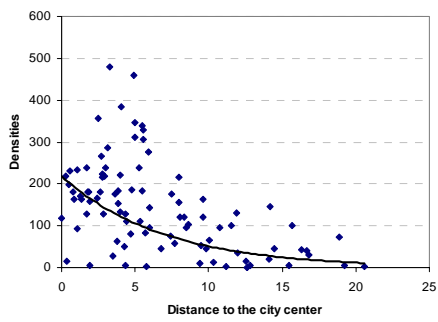
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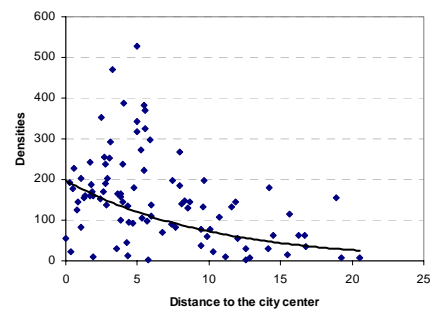
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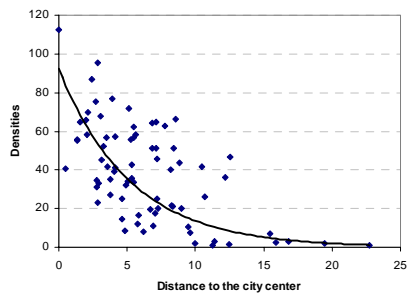
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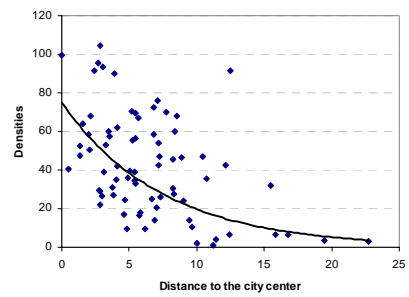
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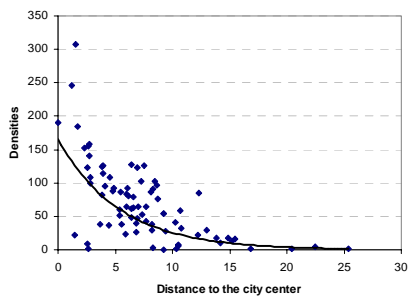
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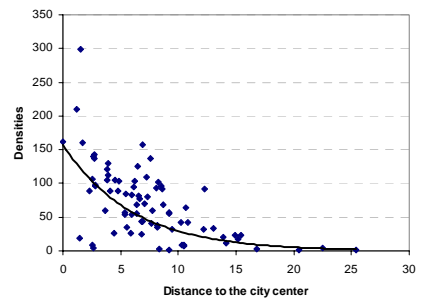
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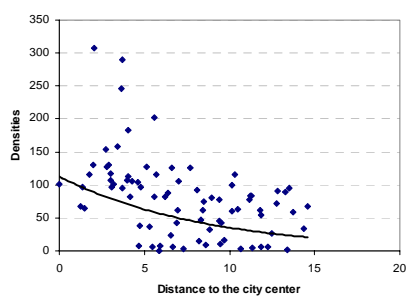
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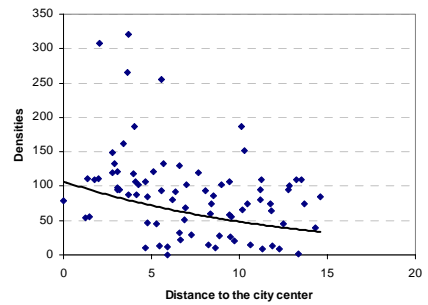
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**BELO HORIZONTE 1991**



**BELO HORIZONTE 2000**



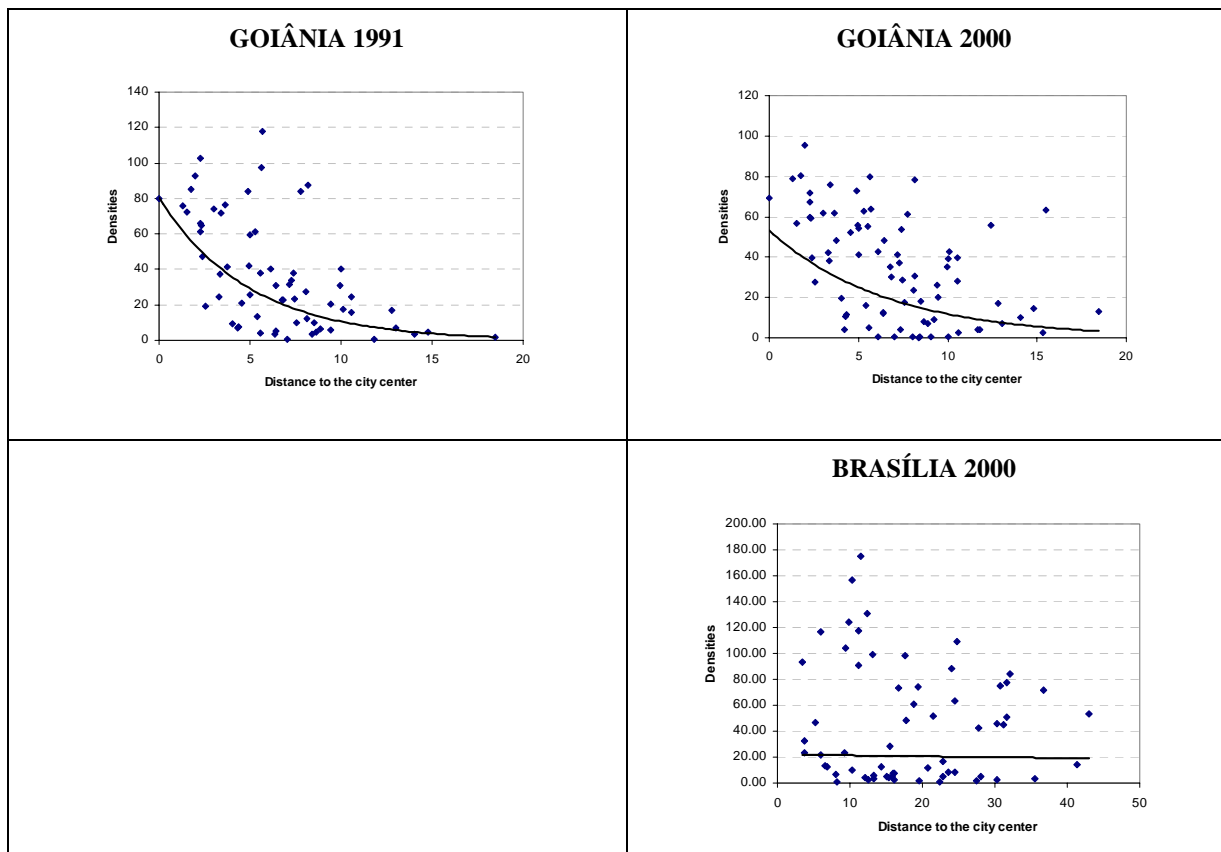


Figure 1 – Population Density Gradients.

Actually, the pattern of decreasing density with distance from the CBD found in the Brazilian cities of our sample is similar to what researchers have observed in most cities in the world. We note that in 2000 the most centralized cities were Porto Alegre and Goiânia, as those presented the highest density gradients among the cities analyzed. These cities have similar area – around 380 km<sup>2</sup> – and a population between 1 and 1.4 million. The city of Recife is comparatively smaller than the former cities in terms of its area, but presented a more decentralized distribution of its population since its population density gradient is smaller. This is probably due to the geography of the city, cut by the canals of the Capibaribe River near the center of the city, increasing the occupation of areas further from the city center.

Note that the gross density of Recife's area is greater than the densities of Porto Alegre and Goiânia, reaching a value more than twice the gross densities of both those cities. This is to be expected since the Recife's area is smaller than the others. However, though smaller size and larger population can mean a more compact city, the Density Gradient found for these three cities

shows that Recife, comparatively, has a more dispersed distribution of its population across its area. Despite this, Recife is the city that has the lowest average distance per person to the city center, of about 6.8 km.

Curitiba is the third most centralized city in our sample, having a few more than 1.5 million people distributed in 434 km<sup>2</sup> area and gross density is of about 36.5 persons per hectare. This is nearly half the density of Fortaleza, a city that has 2.1 million people distributed in a smaller area, about 330 km<sup>2</sup>. In spite of the fact that Fortaleza is a more compact city, with smaller area and bigger population, Curitiba is more centralized than Fortaleza, as shown by its higher density gradient.

The examples of Curitiba and Recife are important points because they indicate that density gradient does not depend on city size or population, but on the pattern of the population distribution in the city area. In general terms, the pattern of population distribution in Curitiba results in a lower average distance per person from the city center than in Fortaleza, what signifies lower transport costs and higher efficiency in the urban services supply. Indeed, the average distance per person in Curitiba is 7.6 km, while in Fortaleza it is 8.0 km per person.

Rio de Janeiro and São Paulo, the largest cities of our sample, presented high average distances per person to the city center, of 19.9 km and 14.1 km, respectively. However, Brasília presented the highest value of our sample, of 20.1 km per person reflecting the dispersion of its urban areas across the territory of Federal District (DF).

In 2000, the more decentralized cities were Brasília, Rio de Janeiro, Belo Horizonte and São Paulo, with the last three as the largest cities in our sample. Despite the reliance of the density gradient on the distribution pattern of the population in the built-up area, big cities, in general, tend to be more decentralized than the small ones. This can be attributed to the rise of agglomerative economies in new areas of the city and the formation of subcenters that share with the Central Business District (CBD) the power in polarizing the occupation in a whole city. This reason cannot be attributed to Brasilia's case, since its polinuclear spatial arrangement has no relationship with subcenters formation, as we will see later.

São Paulo, the largest city in Brazil is more centralized than Rio de Janeiro, a smaller and denser city. This reflects the specific topographic conditions of Rio de Janeiro's site and the attractiveness of the sea coast as location of housing and commerce, a factor that diminishes the

attraction of central areas. Both cities have several subcenters in its built-up area, but the special conditions of the Rio's site yield a more decentralized city.

Belo Horizonte, in spite of its rough topography, similar to the Rio de Janeiro's, though slightly less dramatic, is more centralized. The decentralization of the population in Belo Horizonte between 1991 and 2000 was stronger than decentralization in larger cities such as São Paulo and Rio de Janeiro. The higher population growth rate of Belo Horizonte in the period can explain this fact. Note that the urban growth in Brazilian cities is more intensively absorbed by peripheral areas.

Brasilia, which will be analyzed separately, presents a density gradient curve practically flat, with a slight negative slope. But if some aspects of its spatial structure are considered separately its density gradient inverts and presents a positive slope. The results shown in Table 5 reveal that while the estimated density gradient found for Brasilia is negatively sloped it has no statistical significance. The  $R^2$  also demonstrates that the endogenous variable deployed to explain the density distribution of the city, e.g., the distance to the central areas, doesn't effectively explain the pattern of population distribution across the city. This pattern is a product of the state land market arbitrage combined with strong land use regulations that yield high densities far from the central area of the city, while the city center presents low densities.

Table 5 shows that most of the cities analyzed present an average distance to the city center per person around 7.5 km, except for the largest cities of our sample and Brasilia. This value is about the same of the median distance per person found in cities in other countries. For example, in 1993 Paris had a median distance per person of 7 km while in Moscow that median was about 10 km (Bertaud and Renaud, 1997). The larger median found for Moscow reflects some specific features of its spatial structure, as the misallocation of land near the central areas that exacerbate the increase of the median distance per person to the city center.

Figure 2 show that within 5 to 9 km radius from the city center most Brazilian cities accumulate the major share of their population. This explains average distance per person to the city center of about 7.5 km, in median, found in seven cities of our sample. The exceptions: São Paulo, Rio de Janeiro and Brasília presented different pattern of cumulative population across their urban areas. Brasilia, due to the fragmentation of its spatial structure and the former due to the size of their urban areas.

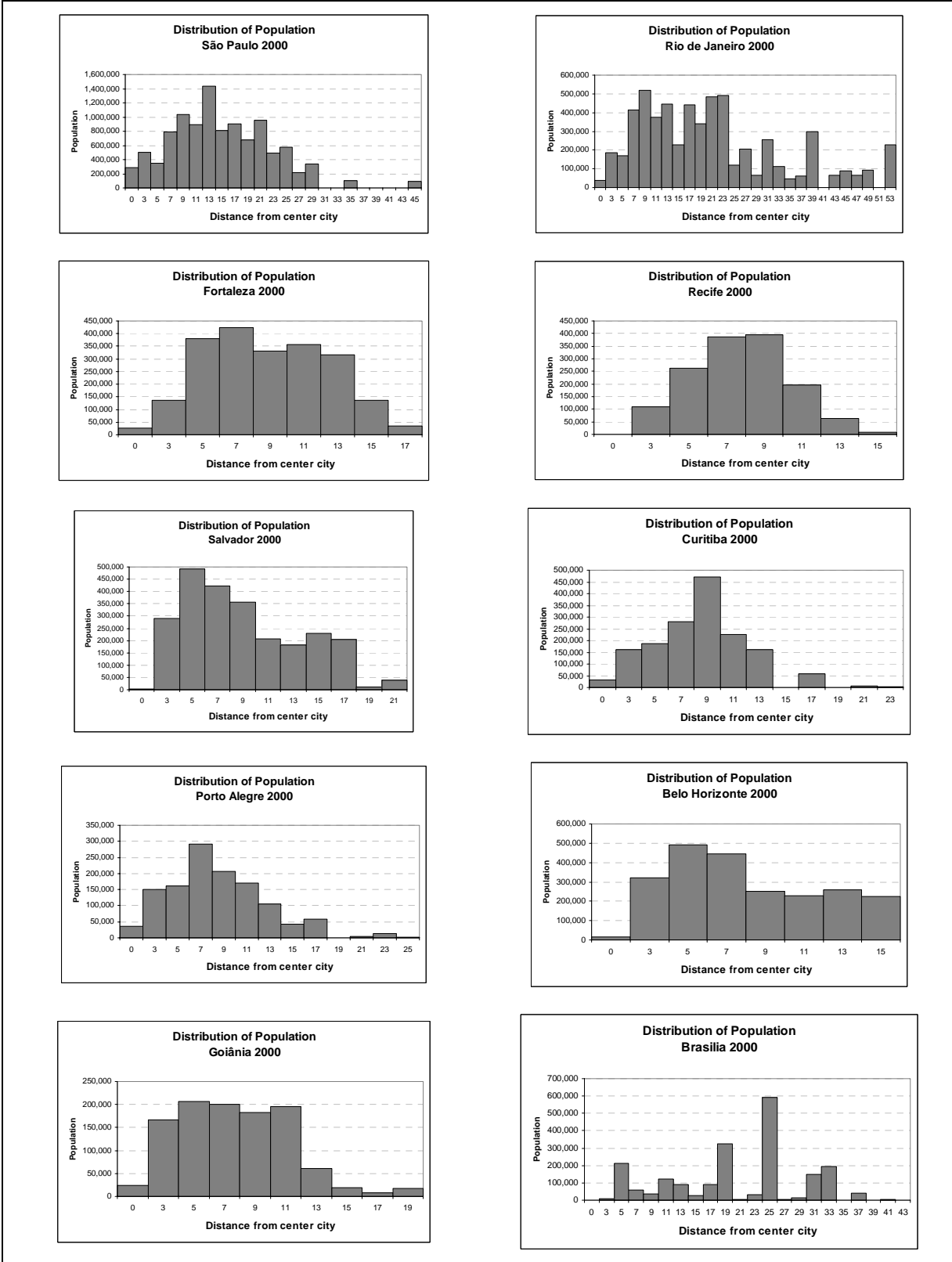


Figure 2 – Population in the Built-up areas of 9 Brazilian cities.

Indeed, the share of the population that lives in the ring within 5 to 9 kilometers radius from the city center corresponds from 52% to 55% of the total in Salvador, Belo Horizonte, Fortaleza, Porto Alegre and Goiânia. In Recife the share of the population that lives at that area corresponds to 73% of the total. In São Paulo and Rio de Janeiro, that population is about 20% of the total, while in Brasilia the population that lives at the same distance from the city center is only 15% of its total. In general, all values measured for Brasília are close to the values reached by the cities that are the core of large metropolitan areas, despite the fact that Brasilia has one fifth of the population of São Paulo and less than half of the population of Rio de Janeiro.

### **III – Brasilia and its spatial pattern of population distribution – Is the Functional City an efficient city?**

Brasília is a city *sui generis*. Built to be the capital of Brazil, its urban plan was chosen in a public contest in 1957 and its construction took only three years. It was inaugurated in 1960, when the administrative staffs of the federal government started to move from Rio de Janeiro, the capital of the country for almost 200 years (1763 – 1960). The winning urban plan, designed by the architect Lucio Costa, projected a city that is tied to the principles of modernism, an architectural movement that adopted a specific conception of architecture and urban planning. Among the principles adopted by the movement's followers was the idea that the cities of the modern industrial era should be organized according to functional areas, connected by axes of transportation that would allow fast linkage among them. The principles for designing the "Functional City" were compiled during the IV International Congress of Modern Architecture (CIAM), held in 1933, in the document named Athens Charter. This document pointed out the four functions that were considered the key issues for the Functional City: to reside, to work, to recreate and to circulate.

The project of moving the Brazilian capital to the hinterland was first proposed in the early 19th century and came to fruition after a complicated political and intellectual itinerary. The move only became a reality when in the mid fifties the candidate for the presidency of Brazil, Juscelino Kubistchek, made a commitment to transfer the capital if he were elected. Thus, the construction of the new capital was added as a synthesis of his government broad goal of transforming the country into a developed and industrialized economy.

At the time, Brazil already had positive experiences in building new cities. Belo Horizonte, projected and constructed to be the capital of the state of Minas Gerais, rose in 1897. Goiânia, officially inaugurated in 1942, was constructed to be the capital of Goiás, the state in which the new capital of Brazil would also be located. Differently from Brasília, none of them were conceived according to modernist postulates and cannot be considered functional cities. Belo Horizonte and Goiânia, in spite of being "new" cities, were not developed under the extreme land use control necessary to keep the division of the city in exclusive functional zones. Despite the existence of plans for the localization of administrative, commercial and residential areas in those cities, the possibility of mixing activities in specific urban areas allows the development of a competitive land market. A land market developed under a competitive basis is the key to efficient land allocation in the absence of economies of scale, public goods, externalities, monopoly control of scarce resources and spatial and temporal path dependence.

In order to build the new capital, the government dispossessed great extensions of farm land, placing in state hands most of the land in the Federal District. Thus, the public administration has been the main agent allocating land for the city development, including land for housing, agricultural, industrial and commercial activities. In addition, the urban developments must obey guidelines traced by Lucio Costa's project and does not allow for changes in the predicted land use determinations. These determinations were stronger within the area of *Plano Piloto*, the area of the original city project and where major job opportunities are concentrated.

Since its inauguration Brasilia attracted an intense migratory flow of people from the states of Goiás and Minas Gerais, and from the poorest areas of the north and northeast regions. In the absence of an affordable housing supply, migrants organized a great number of irregular settlements across the area nearby *Plano Piloto*. Other irregular settlements also started life as construction camps that were not removed after the construction ended.

Some residential areas at the periphery of Brasilia within the area of Federal District (DF) were laid out to provide low cost housing not only for the people that inhabited the slums, but also for administrative workers. In general, the addition of new residential areas, especially those not anticipated by the original plan, was placed by the government far from *Plano Piloto*. Most of

these settlements were supported by public financial resources from the BNH, the National Bank of Housing.

During the 60s and the 70s several plans were produced to control and organize the occupation of the Federal District territory. Normally, these plans and the official housing policies had the intention to control the occupation of *Plano Piloto* surroundings and did follow some recommendations to avoid increasing density in areas near the central areas for environmental and infra-structure reasons. Certainly, the urban areas of Federal District are the most "planned" areas in the urban scene in Brazil. Since its inauguration the several plans that succeeded each other have had in common the aim to preserve the *Plano Piloto*.

These efforts culminated in 1987, when Brasília is assigned as World Cultural Heritage Site by UNESCO. With little more than 37 years of existence Brasília had its principles of spatial organization preserved as heritage of humanity, the only contemporary city to receive this title. The principles of conception of Brasilia are summarized in the 4 scales on the basis of in which the city organization was conceived: the Bucolic Scale, formed by the green and empty spaces that surround functional areas; the Gregarious Scale, formed by the spaces that are set to work activities; the Monumental Scale, the spaces that form the administrative core of the city, and the Residential Scale, the housing areas of the *Plano Piloto*. According to these guidelines, the relationship between the built and empty areas would be preserved, protecting the available areas from new development. As the height of the buildings is fixed, the land recycling via demolition and re-building to construct new and higher structures in response to the increase and spatial variation of land price is obstructed.

During the 80s and 90s, Brazil's economic outlook changed. Due to accelerating inflation, rising internal and external deficits and economic recession, easy public financial support for housing ended and the BNH closed its doors. This conjuncture paired with rigid regulation of land use decreased the supply of housing in Brasilia, especially for the middle classes. However, the robust demand for housing induced the emergence of a private land market that not only traded the undeveloped private land not already dispossessed, but also encouraged irregular and illegal occupation of public land that had not been supplied by the public administration. In short, an "informal" land market emerged.

Since then, many condominium of detached houses were sub-divided and built by private developers operating in an informal land market, driven by low and middle income families without the explicit formal approval of authorities. This so-called "condominium" problem has been the subject of extensive discussion, several studies and an evolving set of policies.

The brief sketch presented above summarizes the historic background of spatial structure of Brasilia, shown in Figure 3. In the first place, we observe how fragmented the spatial structure of the urban area of Brasília is, reflecting, among other things, the official arbitrage in land supply. In addition, since the 80s, several requirements associated to heritage and environmental issues have imposed several constraints to land development. Generally, the undeveloped areas surrounding *Plano Piloto* and the spaces between the areas that form the urban spatial structure of Brasilia have some kind of environmental restriction to their occupation.

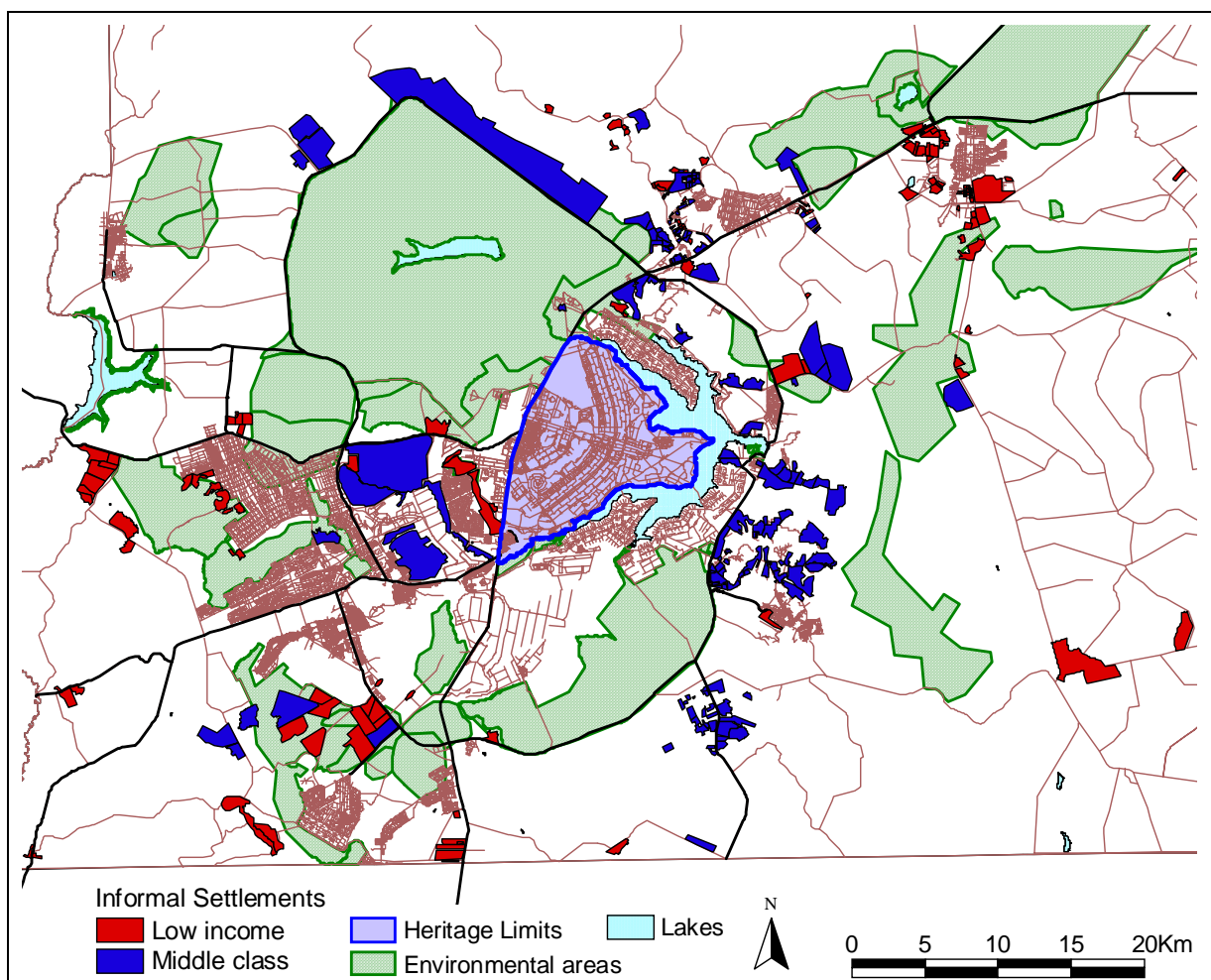


Figure 3 – Urban structure of Brasilia.

In terms of spatial occupation the urban areas of Brasilia are within a 43 kilometers radius but, effectively, its built area corresponds to 598 km<sup>2</sup>, about 10% of the area defined by that radius. Hence, if we consider only the built-up area, the effective density rises from 3.5 to 33.5 persons per hectare. The low density and great distances found in the Federal District result in large transport, infra-structure and amenity costs that are paid by the whole society. Nevertheless, these costs are disproportionately borne by the poor people who live in the most distant areas of the city.

While the population of Brasília is sprawled within a 43 kilometers radius, job opportunities are heavily concentrated in the central areas. Accordingly to Ministry of Labor (MTE) data, about 70% of jobs are concentrated in the central areas, within the heritage perimeter. Hence, daily commuting distances and costs are quite significant, since most of the population lives far from the city center. As a consequence, public transportation in Brasilia is one of the most inefficient in Brazil. While the median of passenger transported by public transportation is between 1.8 and 2.5 per kilometer in cities like Recife, Curitiba and Porto Alegre, in Brasilia it is less than one, reaching only 0.82, accordingly Transportation Ministry data. This indicates that the system in Brasilia is more costly to operate, reflecting the high monetary cost of the public transportation. In addition, with an average distance per person of about 20.1 kilometers, the opportunity cost of the time involved in commuting is very high, even for poor people. This is one of the reasons why the use of cars in Brasília is so intense. In Brasilia there is one vehicle per 2.7 persons, one of the highest rates in Brazil.

Histograms derived from Figure 2 show that in Brasília most of the population is located between 19 and 25 kilometers from the city center, while in the others cities in our sample most people are between 5 and 9 kilometers from central areas. In fact, the plots shown in Figure 4 present the cumulative population across the urban area in those cities. The plots show that, while Brasília cumulates 278,364 people within a 7 km radius of the city center, cities like Belo Horizonte, Salvador and Fortaleza, also with a total population above 2 million, aggregate 1.2 million people in the first two cities and 0.9 million in the last, at the same distance. The 7 km radius in Brasilia includes the residential areas within the heritage limits. If the radius broadens to 9 km, Brasilia presents a cumulative population around 312,487, what corresponds only to 15.6%

of its total, while most of other cities present cumulative population that varies from 60.5% in Fortaleza, with 1.2 million people, to 81% in Recife, with 1.15 million people.

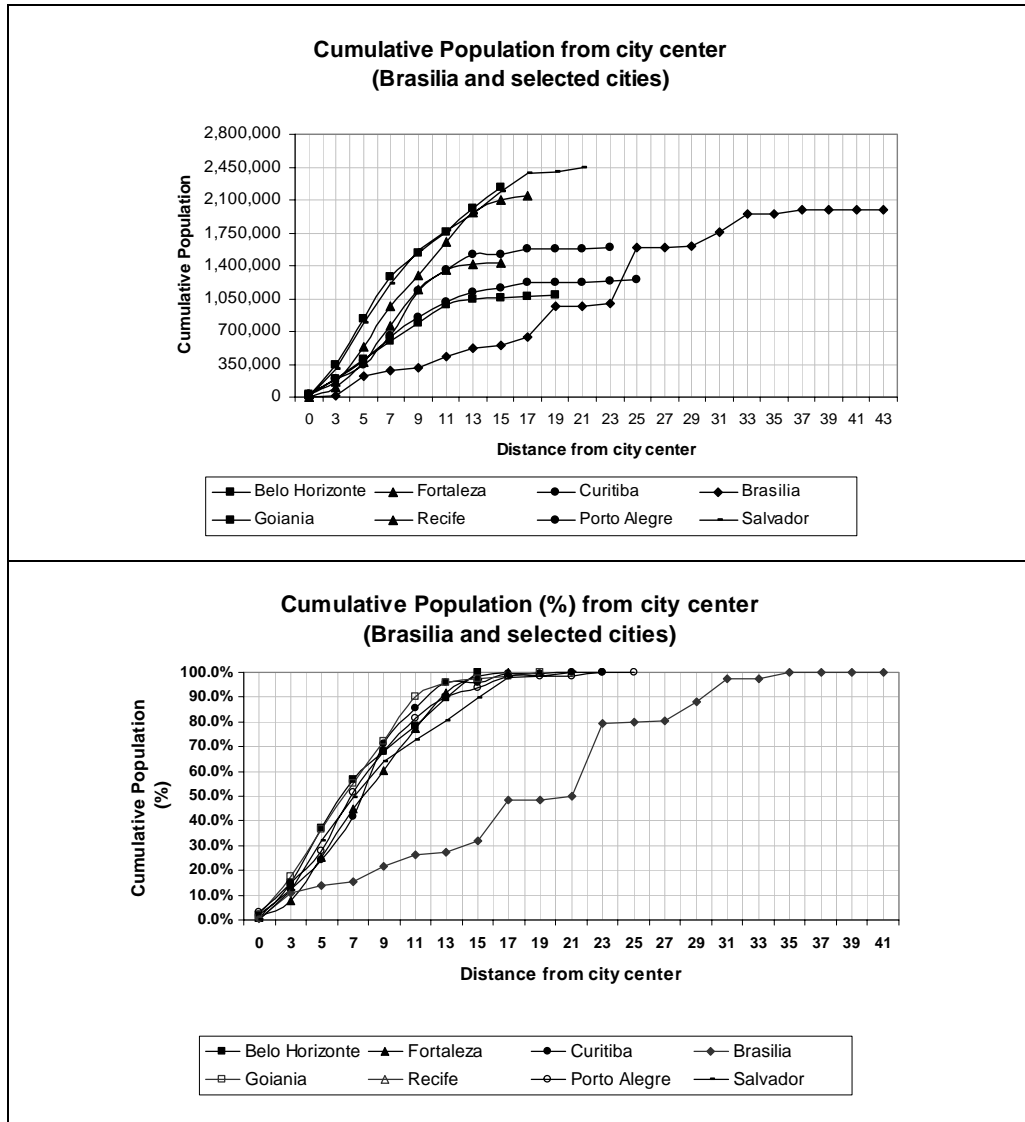


Figure 4 – Cumulative population from the city center in 2000.

The distances in Brasilia are only comparable to those in Rio de Janeiro and São Paulo, even though with a lower population density. Figure 5 shows that within a radius 9 km from the center Brasilia has 312,487 people, while in São Paulo there are 2.9 million and Rio de Janeiro

has 1.3 million people. Nevertheless, these cities present cumulate population less than 30% of the total of their population due to the amount of their populations and their built-up areas.

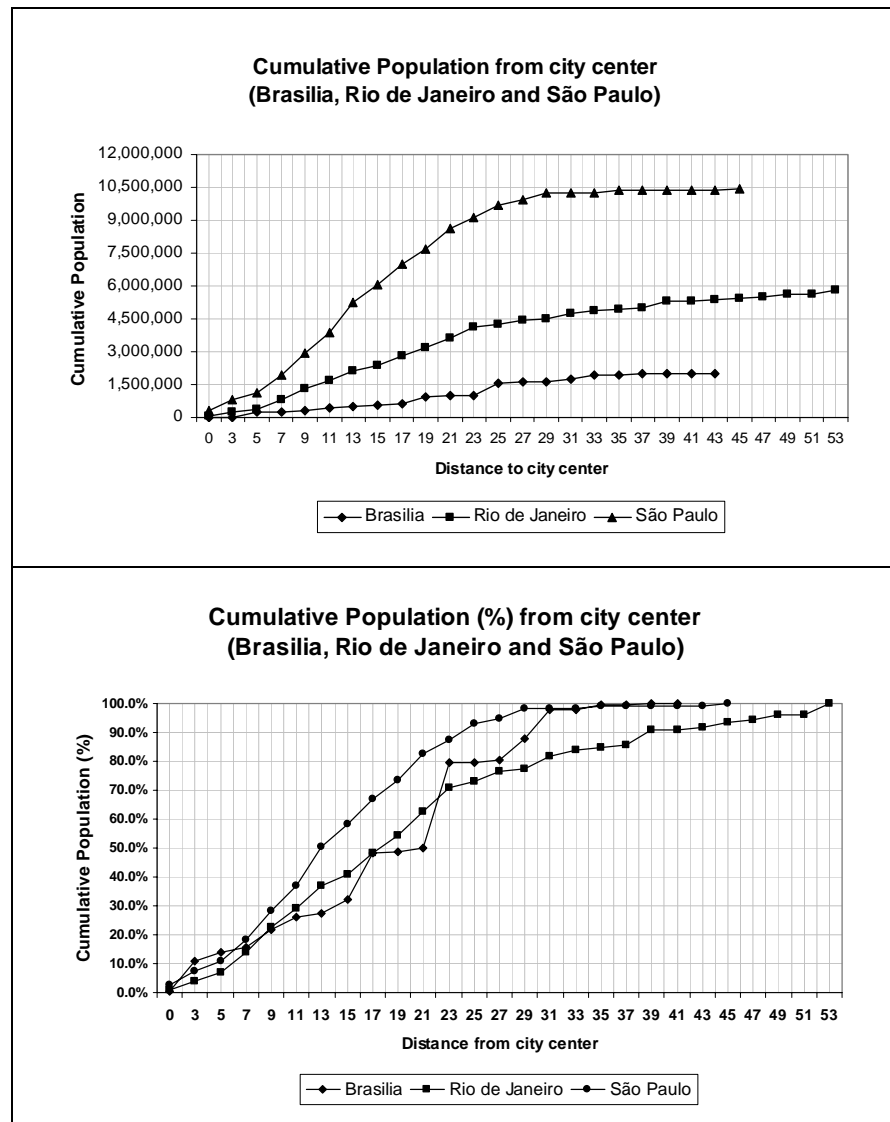


Figure 5 – Cumulative population from the city center in 2000.

Figure 1 showed that the Density Gradient of Brasília is negatively sloped, as predicted by the theory, but practically flat. To deepen our analysis, we made an additional set of estimates of the Population Density Gradient for Brasília considering what we call the "Formal City" and the

"Appropriated City". In the first we considered 32 areas that form the regular and official city. In these areas we included the *Plano Piloto* (and its heritage limits) and the settlements that were developed by local authorities. The second set of regression calculations considered all the areas of the city, including those developed by informal land markets. The areas originating from informal land markets include low income and middle class settlements. Most of these areas were sub-divided in private and public land near the main transportation corridors and near some consolidated urban areas that are located relatively far from the CBD. The results are shown in Figure 6 and respective Tables.

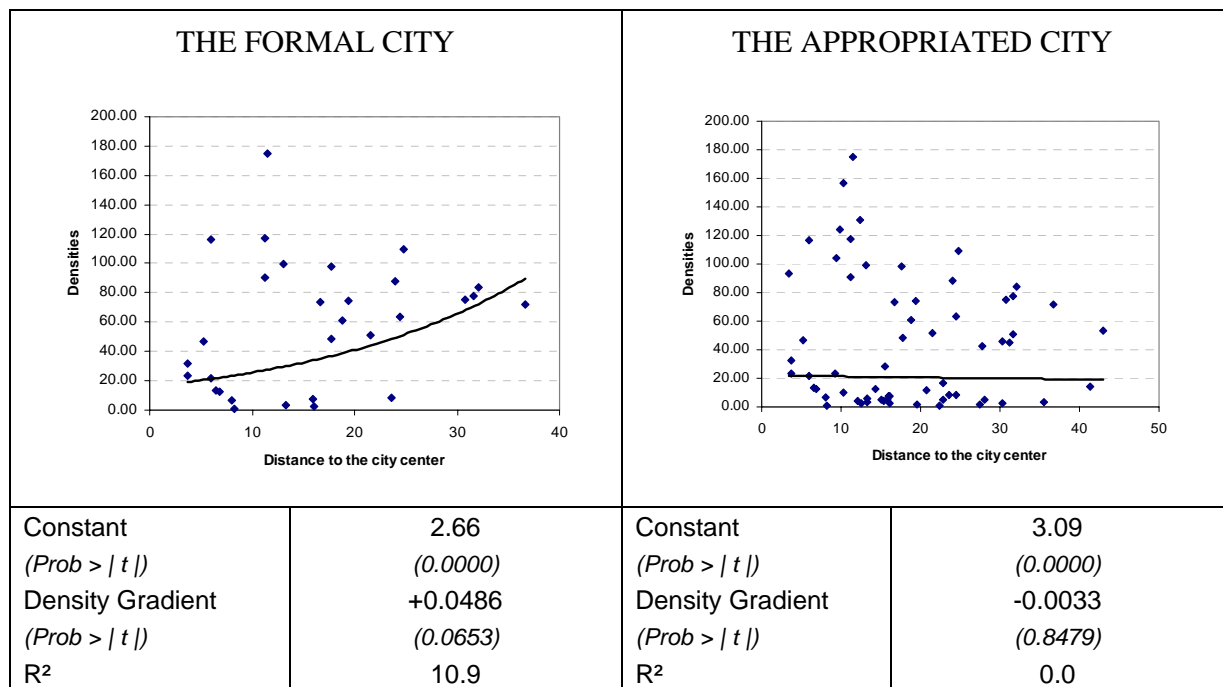


Figure 6 – Population Density Gradients for Brasília in 2000.

Note that in the first regression, e.g., in the Formal City, the Density Gradient curve does not fit the data very well, since the R<sup>2</sup> shows that only 10.9% of the total variation in density is accounted by variation in the distance from the city center. In the Formal City, the location of population was not driven by decentralized mechanisms of choice that take into account proximity to the more accessible areas of the city, mainly the central areas where most jobs are concentrated. On the contrary, the strategy adopted by the authorities of Brasilia was to settle people far from *Plano Piloto*, according to planning guidelines concerned with environmental and urban preservation. Hence, the positively sloped density gradient curve fits well the pattern

of population distribution and the underlying housing policies adopted over the course of time. Nonetheless, the correct specification for a model that would fit data better should incorporate other variables, such as proximity to transport and employment areas, amenities and the rigor of land use regulation, and not only distance from the CBD.

This becomes clearer when we observe the outcome of the second regression, for the Appropriated City, where the Density Gradient curve has a slight negative slope. In this case the  $R^2$  is zero, which signifies that the distance to the city center cannot explain the variation in the dependent variable, the population density. In other words, even though the pattern found corresponds to theoretical predictions of decreasing population density with distance from the city center, the regression line is practically horizontal. In this case, the regression coefficient is zero and the values of the densities can not be predicted by variations in distance to the CBD. Hence, the variation in distance is not as good predictor of densities as in the former case.

Nevertheless, the inversion of the slope of the density gradient when informal settlements are included in the regression calculations demonstrates that these settlements soften the trend of isolating people from the central areas. The spatial pattern where most population is far from the central areas is resulted from state control over the land supply and the housing policies implemented by the government over the time. We consider that the decentralized land market is an efficient arbitrage mechanism to allocate land to the higher and better uses if compared to the command economy. The bad new is that, even considering the informal settlements, poor people continue to live far from the main job locations.

The maintenance of low densities in the central areas is costly and inefficient from the economic land allocation point of view. The opportunity cost involved in maintaining the well-located land undeveloped instead of allocating it for alternative and more efficient use have pushed housing prices higher across the whole city. The increase of the land and housing prices is noted not only in the central areas, but also at the further new areas developed to respond to the housing demand. One of these areas, Aguas Claras, for example, located 20 km from the city center, has experienced a fast increase in its housing values of up to 40% during 2004. Moreover, the high housing prices are responsible for pushing the poor population further away from the center of the city. It seems unlikely that the green-space and amenity benefits of the vast undeveloped areas could offset the opportunity cost derived from its fragmented spatial structure.

The high land prices found in Brasilia suggest that the city, in comparison with others, probably employs capital inputs intensively, perhaps excessively so, in housing production. This reflects the high price per unit of floor area found in Brasilia, higher than in other cities. It is interesting to note that the high land prices that trigger the construction of vertical structures, i.e., inducing substitution and causing higher densities within the central areas in other cities, in Brasilia operates within structures. Since the height of buildings is limited due to constraints imposed by heritage preservation, developers have produced more and smaller housing units per block and unit of land. Hence, one cost of maintaining the city's original spatial conception is that households live in smaller spaces not only in condominium buildings, but also in commercial areas, where offices are also used as housing.

Different spatial arrangements impose different pattern of costs and benefits on the city as a whole. The maintenance of undeveloped land nearby and within the central areas due to land regulations should be evaluated according to cost and benefit considerations and not only according to architectural aspects. While it is unquestionably true that green spaces in the core of the urban area are positive for the quality of life, due consideration should be given to its cost, since this quality of life is appropriated by a small segment of the population.

#### **IV – Conclusion**

The importance of the study of density gradient patterns lies in the help it provides in formulating efficient public policies in urban areas. In sprawling cities, with low densities, the supply of services and infra-structure is costly compared to more compact cities. On the benefit side, cities that are too dense are exposed to diseconomies entailed by externalities like air pollution and traffic jams. The long- term study of densities, linked to the study of land use, can provide elements for understanding the land market dynamics and the development of urban land-value, which in turn is tied to the social stratification of the city space.

Even though the differences that distinguish the birth and the growth of cities located in different regions of Brazil, including the fact the country presents great regional disparities, the spatial structure of its largest cities concerning population distribution across their urban areas is similar among them and among cities worldwide. According to theory, urban population densities decline from city center toward the outer limits of the city due to high land prices at more

accessible sites that trigger the increase of built floor area per land area. By estimating the population density gradients for ten Brazilian cities this pattern was found in nine of them, with the exception of Brasilia, where the expected pattern was inverted.

We found two different results for the population density gradient in Brasilia, positively sloped if considered only the "Formal City" and slightly negative if formal and informal settlements were considered. These findings are emblematic not only of the effects of land allocation commanded by bureaucratic decision and assisted via strong controls over land use, but also the positive effects of the decentralized land market in allocating land for the more efficient uses.

Behind the Population Density Gradients found for Brasilia there is a city whose population is dispersed in an area approximately twice as large as those cities with approximately the same total population. The median distance per person of above 20 kilometers found in Brasilia, for example, shows the dramatic distribution of much of its population at the outer limits of the city. While a small share of the population enjoys the advantages of maintaining low densities in central areas, low income families living on the fringes of the city do not proportionately take advantage of possible benefits of living far from the city center, such as larger houses and a better environment.

The outstanding urban design of *Plano Piloto* and the spatial arrangement of the city as a whole have resulted in high land prices and high intra-urban, out-of-pocket and time travel costs. The supply of primary infra-structure also involves high costs due to large distances involved, exacerbated by extensive empty spaces between urban areas. Probably, the costs derived from that spatial arrangement hinder the competitiveness of the city, private investments and development in general.

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